CLASSICAL AND QUANTUM COMPUTING

EXERCISE IX

We use (Ons maak gebruik van) $\{|0\rangle, |1\rangle\}$ as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Determine the eigenvalues and eigenvectors of (Bepaal die eiewaardes en eievektore van)

$$A(x) := (1-x)I + xU_{NOT}, \qquad x \in \{0,1\}.$$

Show that the unitary transform (Toon aan dat die transformasie)

$$U_f = |0f(0)\rangle\langle 00| + |0\overline{f(0)}\rangle\langle 01| + |1f(1)\rangle\langle 10| + |1\overline{f(1)}\rangle\langle 11|$$

where (waar) $f : \{0,1\} \to \{0,1\}$ is a Boolean function ('n Boole funksie is), can be written as (kan soos volg geskryf word)

$$U_f = |0\rangle\langle 0| \otimes A(f(0)) + |1\rangle\langle 1| \otimes A(f(1)).$$

Calculate (Bereken)

$$U_f \left(I \otimes \frac{1}{\sqrt{2}} \left(|0\rangle - |1\rangle \right) \right).$$

Consider the cases $f(0) = f(1)$ and $f(0) \neq f(1)$ (Beskou die gevalle $f(0) = f(1)$ en $f(0) \neq f(1)$).

2 a) Let U be a unitary operator which acts on two qubits. (Laat U 'n transformasie, wat op twee qubits werk, wees.) Let $|\psi\rangle$ and $|\phi\rangle$ be the states of two arbitrary qubits. (Laat $|\psi\rangle$ en $|\phi\rangle$ twee algemene qubits voorstel.) Determine the implications of measuring the first two qubits of (Bepaal die gevolge van meting van die eerste twee qubits van)

$$|\psi\rangle \otimes \frac{1}{\sqrt{2}} (I \otimes U) \left(|00\rangle + |11\rangle \right) \otimes |\phi\rangle$$

with respect to the the Bell basis (met betrekking to die Bell basis). How can we obtain $U|\psi\rangle \otimes |\phi\rangle$ as the last two qubits ? (Beskryf 'n metode om die laaste twee qubits in die staat $U|\psi\rangle \otimes |\phi\rangle$ te vind.)

2 b) Alice has $|\psi\rangle$ and Bob has $|\phi\rangle$. (Alice het $|\psi\rangle$ en Bob het $|\phi\rangle$.) Describe how U can be applied to $|\psi\rangle \otimes |\phi\rangle$ using only classical communication and prior shared entanglement. (Beskryf hoe U aan $|\psi\rangle \otimes |\phi\rangle$ toegepas kan word, deur net van klassieke kommunikasie en verstrikking wat vooraf beskikbaar is te gebruik.) After the computation, Alice must still have the first qubit of $U|\psi\rangle \otimes |\phi\rangle$ and Bob must still have the second qubit of $U|\psi\rangle \otimes |\phi\rangle$. (Na die berekening, Alice moet die eerste qubit van $U|\psi\rangle \otimes |\phi\rangle$ hê en Bob moet die twede qubit van $U|\psi\rangle \otimes |\phi\rangle$ hê.)