CLASSICAL AND QUANTUM COMPUTING

EXERCISE IV

1) Find the Fourier transform F for (Vind die Fouriertransformasie F van)

$$f_\alpha(x) = \frac{\alpha}{2}\exp(-\alpha|x|), \qquad \alpha > 0.$$

where (waar)

$$F(k) = \int_{-\infty}^{\infty} f(x)e^{ikx}dx$$

Discuss α large and α small. Bespreek α groot en α klein. Calculate (Bereken)

$$\int_{-\infty}^{\infty} f_\alpha(x)dx.$$

2) Consider the Hilbert space $L_2[0, 2\pi]$. Beskou die Hilbertruimte $L_2[0, 2\pi]$. Let (Laat)

$$g(x) = \cos(x), \qquad f(x) = x\,.$$

Find the conditions on the coefficients of the polynomial (Vind die voorwaardes op die koëffisiënte van die polinoom)

$$p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$

such that (sodanig dat)

$$\langle g(x), p(x)\rangle = 0, \qquad \langle f(x), p(x)\rangle = 0\,.$$

Solve the equations for a_3, a_2, a_1, a_0. Los die vergelykings op vir a_3, a_2, a_1, a_0.

3) Consider the Hamilton operator (Beskou die Hamiltonoperator)

$$\hat{S}_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\,.$$

Find (Vind)

$$\exp(-i\hat{S}_x t/\hbar)\,.$$

Let (Laat)

$$\psi(t=0) = \frac{1}{\sqrt{3}}\begin{pmatrix}1\\1\\1\end{pmatrix}\,.$$

Find (Vind)

$$\psi(t) = \exp(-i\hat{S}_x t/\hbar)\psi(t=0)\,.$$

Calculate the probability to find the particle in the initial state after time t. Bereken die waarskynlikheid om die partikel in die aanvangsstaat na tyd t te vind.