CLASSICAL AND QUANTUM COMPUTING
EXERCISE IX

Given a sequence

$$r(0),\ r(1), \ldots, r(p).$$

Using this sequence we can form the $p \times p$ matrix

$$R := \begin{pmatrix} r(0) & r(1) & r(2) & \ldots & r(p-1) \\ r(1) & r(0) & r(1) & \ldots & r(p-2) \\ r(2) & r(1) & r(0) & \ldots & r(p-3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r(p-1) & r(p-2) & r(p-3) & \ldots & r(0) \end{pmatrix}.$$

The matrix is symmetric and all the elements along the diagonal are equal. We assume that $r(0) \neq 0$ and R is invertible. Let

$$A := \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix}, \qquad P := \begin{pmatrix} r(1) \\ r(2) \\ \vdots \\ r(p) \end{pmatrix}.$$

Then the linear equation

$$RA = P$$

can be solved using Durbin's algorithm which is a recursive procedure. The algorithm is as follows:

$$\begin{aligned} E(0) &:= r(0) \\ k_1 &= r(1)/E(0) \\ k_i &= \left[r(i) - \sum_{j=1}^{i-1} a_j(i-1) r(i-j) \right] / E(i-1), \quad i = 2, 3, \ldots, p \\ a_i(i) &= k_i, \quad i = 1, 2, \ldots, p \\ a_j(i) &= a_j(i-1) - k_i a_{i-j}(i-1), \quad \begin{cases} i = 2, 3, \ldots, p \\ j = 1, 2, \ldots, i-1 \end{cases} \\ E(i) &= (1 - k_i^2) E(i-1), \quad i = 1, 2, \ldots, p \end{aligned}$$

After solving these equations recursively, the solution of the linear equation is given by

$$a_j = a_j(p), \quad j = 1, 2, \ldots, p$$

Write a C++ or Java program that implements this algorithm. Test your program with the sequence

$$r(0) = 1, \quad r(1) = 0.5, \quad r(2) = 0.2$$

This problem plays a role in linear predictive coding.