CLASSICAL AND QUANTUM COMPUTING
EXERCISE VII

1. We have 50 'gold' coins all of the same weight, except for one which is fake and weighs less. In other words we have 49 gold coins of equal weight and one fake coin of less weight. We also have a balance scale. Any number of coins can be put on each side of the scale at the same time. The scale will indicate if the two sides weigh the same, or which side is lighter if this is not the case. Find an algorithm to locate the fake coin using the balance scale as few times as possible. If there are $2n$ gold coins one of which is fake, how many times would the algorithm use the balance scale?

2. Given a set of N non-negative integers (all the integers are pairwise different). Let N be odd. Write a program which uses an array to represent the set. Implement algorithms to find the minimum, maximum and median of the numbers. The median is the number such that $(N-1)/2$ of the numbers are less than the median and the other $(N-1)/2$ numbers ar larger than the median. Also determine the position in the array of the minimum, maximum and median. In each case determine the number of comparisons needed to obtain the result.

   Consider for example the set

   $$\{3, 4, 11, 2, 8, 20, 10\}.$$

   The minimum is 2, the maximum is 20 and the median is 8. The position of the median is 4 if we number the positions from 0, from left to right.