

CLASSICAL AND QUANTUM COMPUTING  
EXERCISE IV

1) Find the Fourier transform  $F$  for (Vind die Fouriertransformasie  $F$  van)

$$f_\alpha(x) = \frac{\alpha}{2} \exp(-\alpha|x|), \quad \alpha > 0.$$

where (waar)

$$F(k) = \int_{-\infty}^{\infty} f(x)e^{ikx} dx$$

Discuss  $\alpha$  large and  $\alpha$  small. Bespreek  $\alpha$  groot en  $\alpha$  klein. Calculate (Bereken)

$$\int_{-\infty}^{\infty} f_\alpha(x) dx.$$

2) Consider the Hilbert space  $L_2[0, 2\pi]$ . Beskou die Hilbertruimte  $L_2[0, 2\pi]$ . Let (Laat)

$$g(x) = \cos(x), \quad f(x) = x.$$

Find the conditions on the coefficients of the polynomial (Vind die voorwaardes op die koëffisiënte van die polinoom)

$$p(x) = a_3x^3 + a_2x^2 + a_1x + a_0$$

such that (sodanig dat)

$$\langle g(x), p(x) \rangle = 0, \quad \langle f(x), p(x) \rangle = 0.$$

Solve the equations for  $a_3, a_2, a_1, a_0$ . Los die vergelykings op vir  $a_3, a_2, a_1, a_0$ .

3) Consider the Hamilton operator (Beskou die Hamiltonoperator)

$$\hat{S}_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

Find (Vind)

$$\exp(-i\hat{S}_x t/\hbar).$$

Let (Laat)

$$\psi(t=0) = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Find (Vind)

$$\psi(t) = \exp(-i\hat{S}_x t/\hbar)\psi(t=0).$$

Calculate the probability to find the particle in the initial state after time  $t$ . Bereken die waarskynlikheid om die partikel in die aanvangsstaat na tyd  $t$  te vind.