CLASSICAL AND QUANTUM COMPUTING
EXERCISE XIII

1) Any integer j ($j \in \mathbf{Z}$) can be written uniquely as

$$j = 2^n(2s+1).$$

Given j, write a program which finds n and s which satisfy the above equation.

2) The LZW algorithm is a general purpose compression scheme developed by Lempel-Ziv and Welch. The algorithm uses a table of 4096 entries to store frequently used patterns of information. Initially the first 256 entries are set to the values a single byte can take on (i.e. each entry i has the value i for $i = 0, \ldots, 255$). Each of the first 512 entries of the table are represented using 9 bits. When the table is filled up to exceed 512 entries then 10 bits are used for each index. Similarly 11 and 12 bits are used for each index when the table exceeds 1024 and 2048 entries respectively. The first 258 table entries are fixed. The entry 256 indicates a *clear code* which indicates all entries above 257 should be cleared. The entry 257 indicates end of information, i.e. the current block of data is completely encoded. The way the table is filled up is as follows

1. `P=""`
2. read `C` (a character)
3. if `P+C` (string concatenation) is a string in the table
   `P=P+C`
   goto 2
4. write the index in the table where P was found
   add `P+C` to the table
   `P=""`
   goto 3

When the table is full (all 4096 entries used) the table must first be cleared before starting anew. Thus the index 256 must be written to the output.

Write a program which encodes data using the algorithm. Write a program which decodes the encoded data.