Integer programming-based constructions of Latin squares

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Integer programming models may be used to construct special types of Latin squares by appropriately defining the variables and constraints such that a feasible (or optimal) solution defines the desired Latin square. Recently Appa *et al.* [1, 2] used integer programming in order to solve certain problems concerning mutually orthogonal Latin squares, including a proof of the non-existence of a pair of mutually orthogonal Latin squares of order 6 by an attempted LP-based construction of such a pair. In this talk we consider the construction a various types of Latin squares by utilising integer programming as well as algebraic identities which define special types of Latin squares. These algebraic identities may be incorporated as constraints in an integer programming model in order to construct a quasi-group for which the multiplication table represents the desired Latin square. One such an example is a self-orthogonal Latin square (SOLS) (a Latin square orthogonal to its transpose), where the multiplication table of a quasi-group satisfying the identity $yx = x(xy)$ represents a SOLS.

References

[1] Appa G, Magos D & Mourtos I, *Integrating constraint and integer programming for the orthogonal Latin squares problem,* pp. 15–31 in Van Hentenryck P (Ed), Principles and Practice of Constraint Programming-CP2002, Lecture Notes in Computer Science, Vol. 2470 2002, Springer, Berlin.
[2] Appa G, Magos D & Mourtos I, *An LP-based proof for the non-existence of a pair of orthogonal Latin squares of order 6,* Operations Research Letters, (32) 2004, pp. 336–344.