Generalizing a hexagon result via proof

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In the majority of textbooks at high school and university, the purpose of proof in mathematics is still presented almost exclusively as that of *verification*; i.e. only as a means of obtaining certainty and to eliminate doubt. However, proving is not just about making sure. Particularly, given the very high level of conviction one can nowadays obtain through many different computer programs, proof may instead serve the purpose of a logical *explanation* of *why* a certain result is true (see De Villiers, 2003). Moreover, a proof can also sometimes serve as a tool for *discovery* since it often provides valuable insight into why a result is true, hence immediately enabling one to generalize or vary the result in different ways. The purpose of this paper is to give one example of a relatively recent problem I worked on that illustrates this *"discovery"* function very well. The investigation started with the following interesting result that was discovered with the aid of *Sketchpad*: If $ABCDEF$ is a hexagon with opposite sides parallel (not necessarily equal), then the respective centroids G, H, I, J, K and L of triangles ABC, BCD, CDE, DEF, EFA and FAB, form a hexagon with opposite sides both equal and parallel.