

Learning to define in geometry

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“Let us remember that mathematics begins with questions, and only ends in definitions.”

- paraphrasing L. W. H. Hull (1969) in “The superstition of educated men”, *Mathematics Teaching*, 43, p. 30.

“the Socratic didactician would refuse to introduce the geometrical objects by definitions, but wherever the didactic inversion prevails, deductivity starts with definitions. The Socratic didactician rejects such a procedure. How can you define a thing before you know what you have to define?” - Hans Freudenthal (1973) in *Mathematics as an Educational Task*, Dordrecht: D. Reidel, p. 416.

The construction of definitions (defining) is a mathematical activity of no less importance than other mathematical processes such as solving problems, making conjectures, generalizing, specializing, proving, etc. It is therefore perhaps a step in the right direction that the current South African curriculum (2005) specifically states in regard to Learning Outcome 3: Shape, Space and Measurement for Grade 10 that the learner should:

- investigate necessary and sufficient conditions to define various polygons (including the isosceles, equilateral and right-angled triangles, the trapezium, the isosceles trapezium, kite, parallelogram, rectangle, rhombus, square, etc.)

But teaching learners, or even students and prospective mathematics teachers at university, how to define geometrical objects is fraught with several difficulties. The purpose of this paper to provide some theoretical perspectives, as well as some research data on the problem. For example, a distinction will be made between two kinds of defining in mathematics, namely, *descriptive* and *prescriptive* defining. Closely associated with defining is the process of classification, and in relation to the quadrilaterals, establishing a hierarchical classification is particularly problematic for many learners. More-over, in any such approach attention should be given, either implicitly or explicitly, to necessary and sufficient conditions, as students often mistakenly regard necessary conditions as sufficient conditions for defining concepts.