Idempotents in Stone-Čech compactifications and homogeneous maximal spaces

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Let G be an infinite group and let βG be the Stone-Čech compactification of G as a discrete semigroup. We take the points of βG to be the ultrafilters on G. Being a compact right topological semigroup, βG has idempotents. Every idempotent $p \in \beta G$ determines a left translation invariant Hausdorff topology $\mathcal{T}_p$ on G with a neighborhood base at the identity $e \in G$ consisting of subsets $A \cup \{e\}$ where $A \in p$. An idempotent $p \in \beta G$ is *regular* if p is uniform (= for every $A \in p$, $|A| = |G|$) and the topology $\mathcal{T}_p$ is regular. We show that for every infinite group G, there exists a regular idempotent in βG. As a consequence we obtain that for every infinite cardinal κ, there exists a homogeneous regular maximal space of dispersion character κ, which is the answer to an old difficult question. Another consequence tells us that there exists a translation invariant regular maximal topology on the real line of dispersion character $\mathfrak{c}$ stronger than the natural topology.