# Transversals in hypergraphs, total domination in graphs and other related problems

**A. Yeo**
Royal Holloway, University of London
*anders@cs.rhul.ac.uk*

A set $S$ of vertices in a graph $G$ is a total dominating set of $G$ if every vertex of $G$ is adjacent to some vertex in $S$. The minimum cardinality of a total dominating set is called the *total domination number*.

A *hypergraph*, $H$, contains a vertex set denoted by $V(H)$ and an edge set denoted by $E(H)$. Every hyper-edge in $H$ is a subset of the vertices. For graphs these sets all have size two, but for hypergraphs they can have any size. A *transversal* (also called a *hitting set*) in a hypergraph, $H$, is a set of vertices $T \subseteq V(H)$, such that every hyper-edge in $E(H)$ contains at least one vertex from $T$.

We will both give bounds on the size of transversals in several kind of hypergraphs and show how these bounds can be used to obtain many different kind of bounds for the total domination number of a graph with properties such as *(i) minimum degree 3 or 4*, *(ii)* 2*-connected*, *(iii) minimum degree 2, containing no induced* 6*-cycles* and *(iv) minimum degree 3, containing no* 4*-cycle*.

The area of *fixed parameter tractable algorithms* is new, interesting and growing rapidly. As finding transversals in 3-uniform hypergraphs (i.e. all edges contain 3 vertices) has many application, we will mention a fixed parameter tractable algorithm for this problem. This algorithm can immediately be used in areas such as computational biology (related to phylogenetic trees) and tournaments (finding a minimum feedback vertex set). The time complexity of our algorithm beats all previously know algorithms.

We finally mention several open problems and conjectures.