Determinant identities for Laplace matrices and their applications

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We show that every minor of an $n \times n$ Laplace matrix, i.e., a symmetric matrix whose row- and column sums are 0, can be written in terms of those $\binom{n}{2}$ minors that are obtained by deleting two rows and the corresponding columns.

This identity has interesting applications to the enumeration of spanning trees. Specifically, we prove that if a subgraph of a graph G is replaced by an electrically equivalent graph, the number of spanning trees only changes by a factor that does not depend on G. This allows us to employ techniques from the theory of electrical networks—such as the Wye-Delta transform—to determine the number of spanning trees of a graph, a technique that is particularly useful if the graphs under consideration are highly symmetric.

Furthermore, the identity also leads to a solution of the following inverse problem: Given an electrical network, the so-called effective resistance between any two vertices can be computed in terms of the given resistances. Our identity yields an explicit formula for resistances in terms of effective resistances.