

# Pseudo-topologies on reduced power algebras

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**SAMS Subject Classification: Algebra; Topology**

We show how pseudo-topologies, in particular convergence structures, may be defined in a natural way on reduced power algebras (RPA-s). As a first and most basic example, we give a construction of the set  $\mathbb{R}$  of real numbers, and of its usual metric topology, as a *quotient space* of a suitable subset of  $\mathbb{Q}^{\mathbb{N}}$ . In particular, we show how the topology on  $\mathbb{R}$  can be obtained as a quotient topology, with respect to a suitable topology on the set of Cauchy sequences from  $\mathbb{Q}$ . This construction serves as the prototype for *pseudo-topologies* on more general, and in particular non Archimedean, reduced power algebras. A major interest in such an extension of the usual concept of topology is in its convenient categorial properties which are unfortunately missing in the case of the usual topology. And such categorial properties turn out to lead in simple natural ways to important topological ones.