

Automorphism groups of generalized triangular matrix rings

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We call a ring strongly indecomposable if it cannot be represented as a non-trivial (i.e. $M \neq 0$) generalized triangular matrix ring $\begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$, for some rings R and S and some R - S -bimodule ${}_R M_S$. Examples of such rings include rings with only the trivial idempotents 0 and 1, as well as endomorphism rings of vector spaces, or more generally, semiprime indecomposable rings. We show that if R and S are strongly indecomposable rings, then the triangulation of the non-trivial generalized triangular matrix ring $\begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$ is unique up to isomorphism; to be more precise, if $\varphi : \begin{pmatrix} R & M \\ 0 & S \end{pmatrix} \rightarrow \begin{pmatrix} R' & M' \\ 0 & S' \end{pmatrix}$ is an isomorphism, then there are isomorphisms $\rho : R \rightarrow R'$ and $\psi : S \rightarrow S'$ such that $\chi := \varphi|_M : M \rightarrow M'$ is an R - S -bimodule isomorphism relative to ρ and ψ . In particular, this result describes the automorphism groups of such upper triangular matrix rings $\begin{pmatrix} R & M \\ 0 & S \end{pmatrix}$.

References

- [1] G. Abrams, J. Haefner and A. del Río, *The isomorphism problem for incidence rings*, Pacific J. Math. 187(2) (1999), 201-214.
- [2] S. P. Coelho, *The automorphism group of a structural matrix algebra*, Linear Algebra Appl. 195 (1993), 35-58.
- [3] S. Dăscălescu and L. van Wyk, *Do isomorphic structural matrix rings have isomorphic graphs?*, Proc. Amer. Math. Soc. 124(5) (1996), 1385-1391.
- [4] S. Dăscălescu and L. van Wyk, *Complete blocked triangular matrix rings over a Noetherian ring*, J. Pure Appl. Algebra 133 (1998), 65-68.
- [5] S. Dăscălescu and L. van Wyk, *The recovery of the non-diagonal tile in a tiled triangular matrix ring*, Indian J. Math. 42(2) (2000), 167-173.
- [6] S. Jondrup, *Automorphisms of upper triangular matrix rings*, Arch. Math. (Basel) 49(6) (1987), 497-502.
- [7] T. P. Kezlan, *A note on algebra automorphisms of triangular matrices over commutative rings*, Linear Algebra Appl. 135 (1990), 181-184.
- [8] R. Khazal, S. Dăscălescu and L. van Wyk, *Isomorphism of generalized triangular matrix-rings and recovery of tiles*, Internat. J. Math. Math. Sci. 2003(9) (2003), 533-538.