

Domination partitions in graphs

J. Southey* and M.A. Henning

University of KwaZulu-Natal

just.so@hotmail.com, henning@ukzn.ac.za

SAMS Subject Classification: Combinatorics and Graph Theory

A dominating set of a graph is a set of vertices such that every vertex not in the set is adjacent to a vertex in the set, a total dominating set of a graph is a set of vertices such that every vertex is adjacent to a vertex in the set, and a paired-dominating set of a graph is a dominating set such that the subgraph induced by the dominating set contains a perfect matching. A simple yet fundamental observation made by Ore [1] is that every graph of minimum degree at least one contains two disjoint dominating sets. In contrast to that, Zelinka [2, 3] showed that no minimum degree is sufficient to guarantee the existence of two disjoint total dominating sets (and hence also, two disjoint paired-dominating sets). In this talk, a number of results are presented which are somehow located between these observations. In particular, we show that the vertex set of every graph with minimum degree at least two and with no component that is a 5-cycle can be partitioned into a dominating set and a total dominating set. Furthermore, we present a constructive characterization of graphs whose vertex set can be thus partitioned. We then contrast similar results for partitioning the vertex set of a graph into a dominating set and a paired-dominating set.

References

- [1] O. Ore, *Theory of graphs*, Amer. Math. Soc. Transl. 38 (Amer. Math. Soc., Providence, RI, 1962), 206-212.
- [2] B. Zelinka, *Total domatic number and degrees of vertices of a graph*, Math. Slovaca 39 (1989), 7–11.
- [3] B. Zelinka, *Domatic numbers of graphs and their variants: A survey*, Domination in Graphs: Advanced Topics, T.W. Haynes et al. eds, Marcel Dekker, New York, 1998, 351–377.