We present a comprehensive deterministic mathematical model for HIV, which includes wild-type and drug-resistant strain. The model consists of a system of ordinary differential equations describing the transmission dynamics of HIV in the presence of antiretroviral therapy (ART). The principle objective is to assess the dynamics of the two strains and identify the key epidemiological factors. We carried out an extensive analytical and numerical analysis. We determined the basic reproductive numbers for each strain ($R_{0R}$ and $R_{0W}$) and perform an uncertainty and sensitivity analysis on the reproduction numbers. The model has a disease free equilibrium which is globally asymptotically stable whenever both reproduction number is less than unity. This asymptotic global stability is shown using comparison theorem. Using the Center of manifold theorem we showed the existence and stability of drug-resistant only boundary equilibrium and wild-type only boundary equilibrium. The result shows that wild-type only boundary equilibrium exist and it is locally asymptotically stable whenever $R_{0R} < 1 < R_{0W}$ and that drug-resistant only boundary equilibrium exist and it is locally asymptotically stable whenever $R_{0W} < 1 < R_{0R}$. The model can have a coexistence equilibrium whenever both reproduction number are greater than unity. Simulation results suggest that treatment rate, risky behavior, rate of development of resistant strain and fitness of resistant strain are key epidemiological factors that either increase or decrease HIV prevalence.
Upper bounds on the Steiner diameter of a graph

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SAMS Subject Classification: Combinatorics and Graph Theory

Let $G$ be a connected graph and $S$ a nonempty set of vertices of $G$. Then the Steiner distance, $d(S)$, of $S$ is the minimum size of a connected subgraph of $G$ whose vertex set contains $S$. If $n$ is an integer, $2 \leq n \leq p$, the Steiner $n$-diameter, $\text{diam}_n(G)$, of $G$ is the maximum Steiner distance of any $n$-subset of vertices of $G$.

In this talk we give an improved bound on $\text{diam}_n(G)$ for a graph $G$ in terms of the order of $G$ and the minimum degree of $G$. Also we give bounds on $\text{diam}_n(G)$ for $K_3$-free graphs and $C_4$-free graphs. Moreover, we demonstrate that the bounds are best possible.
Independent vertex subsets of degree-restricted trees

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SAMS Subject Classification: Combinatorics and Graph Theory

The number of independent subsets of a graph is a parameter that has been studied extensively in the literature. Among many other results, it is known that the paths minimize the parameter among all trees. Motivated by the interesting relation to organic chemistry, we studied the class of trees containing vertices of degree one or four only. Our main result is that the graphs within this class that minimize the number of independent vertex subsets are path-like trees. Generalization in various directions will be discussed.
A mathematical epidemiological model of Bartonella infections in the invasive Rattus rattus in South Africa

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SAMS Subject Classification: Applications of Mathematics to the Sciences

We study mathematically the relationships between the contributing factors of Bartonella infections in a rural rodent population of South Africa. We develop a mathematical model in the form of a system of ordinary differential equations representing a dynamical system involving both the host (Rattus rattus) and the vector (Ixodidae) by using a compartmental approach. Apart from giving a better understanding of the relationships and their significance for the sylvatic cycle of Bartonella, the model may impact on future research direction and the design of control measures. While the modelling process enhances the existing knowledge, it also highlights the lack of it in relationships of this sylvatic cycle.
On the regular semisimple elements and primary classes of $GL(n, q)$

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SAMS Subject Classification: Algebra

In this talk we count the number of regular semisimple elements, together with the number of primary classes of the general linear group $GL(n, q)$. The approach used to count these numbers depends essentially on partitions of positive integers $\leq n$. We list the numbers of regular semisimple elements and primary classes of $GL(n, q)$ for $n \in \{1, 2, \cdots, 6\}$ and see that the number of regular semisimple elements is an integral polynomial in $q$, while the number of primary classes is a rational polynomial in $q$. 
Continued fractions, hyperbolic geometry, and Farey arithmetic.

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Continued fractions are usually first met in number theory, but they can be viewed as the boundary action of a group of isometries of the hyperbolic plane, and as paths in the abstract Farey graph. In this talk I shall give a little of the history of continued fractions, and then show how they arise naturally from hyperbolic geometry, and exist on the boundary of hyperbolic space of any dimension.
In this paper we present a spatially homogeneous locally-rotationally-symmetric (LRS) Bianchi type-V cosmological model with perfect fluid and heat flow. A general approach is introduced to solve Einstein’s field equations by using a law of variation for the mean Hubble parameter, which is related to the average scale factor of the model that yields a constant value for the deceleration parameter. The law generates power-law and exponential forms for the average scale factor in terms of the cosmic time $t$. Exact solutions that correspond to singular and non-singular models are found with heat flow. We find that a constant value of the deceleration parameter is reasonable for a description of the different phases of the universe. The physical constraints on the solutions of the field equations, and, in particular, the thermodynamical laws and energy conditions that govern such solutions are discussed in some detail. The dynamical behaviors of the solutions and kinematical parameters like expansion, shear and the anisotropy parameter are discussed in detail in each section.
Hopf bifurcation of tuberculosis transmission model with vaccination

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SAMS Subject Classification: ODE’s and Dynamical Systems

We consider a deterministic SEI model for a transmission of tuberculosis. The model incorporates vaccination for newborns in which only a proportion of them are vaccinated at birth. We analyse the model mathematically and numerically. Bifurcation of multiple endemic equilibrium points exist as we varied a reinfection term. For this, a threshold value of the reinfection rate is set resulting in different kinds of bifurcation. Thus the system exhibits a backward and forward bifurcation having hopf bifurcation (as result of inherited periodic solutions) depending on the reinfection rate chosen. For certain chosen set of parameters, two endemic equilibriums exist below $R_0 = 1$, in which neither of them are stable. Thus it can be suggested that, disease free equilibrium (DFE) is globally stable for $R_0 < 1$. This result is different compared to the usual backward bifurcations, as the co-stability of the disease free and endemic equilibrium points has been discussed widely in other studies. While for $R_0$ above unity, the system has three endemic equilibrium points up to certain value of $R_0$ and onwards only one. From the three, only one endemic equilibrium point is stable and the DFE is unstable. The numerical results also reveal that, vaccination is of less importance if the waning effect is comparably high. It implies that treatment has to target unvaccinated individuals or boost vaccination has to be used. This is mainly because of high transition of vaccinated individuals to the respective unvaccinated compartments as protection of the vaccine wanes.
The quasi-normal modes of a white hole

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SAMS Subject Classification: Mathematical Physics

We linearize the Einstein equations about the Schwarzschild geometry within a Bondi-Sachs, or null cone, framework. The quasi-normal modes are those frequencies at which the perturbations are regular at both the event horizon and infinity. We develop a numerical method to find the quasi-normal modes for this problem, and present results in the simplest case, $\ell = 2$. The results obtained are different to the well-known values of a Schwarzschild black hole. In the present case the hypersurface on which the calculation is performed intersects only the past horizon and not the future horizon, and therefore the results obtained represent the quasi-normal modes of a white hole.
Soliton perturbation theory for phi-four model and nonlinear Klein-Gordon equations

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SAMS Subject Classification: Mathematical Physics

This talk is on the study of adiabatic variation of the soliton velocity, in the presence of perturbation terms, of the phi-four model and the nonlinear Klein-Gordon equations. There are four types of models of the nonlinear Klein-Gordon equation, that will be talked about. The soliton perturbation theory is utilized to carry out this investigation.
The NBT and the NSC: how to optimise the partnership

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SAMS Subject Classification: History and Teaching of Mathematics

For many years various assessment tools have been evident at the post-school, pre-university level, such as the matriculation examination (pre 2008), the NSC (from 2008) and the Alternative Admissions Research Project (AARP) tests used by many universities. At the end of 2008 school leavers wrote the new National Senior Certificate (NSC) examination for the first time. From 2005 the National Benchmark Test Project (NBTP) has focused on the development of national tests in academic/quantitative literacy and mathematics, piloted in 2008 (small scale) and in 2009, culminating in a standard setting exercise in 2009, which determined two benchmarks for these domains. The lower benchmark separates students unlikely to succeed in Higher Education (HE) programmes from those likely to succeed provided they receive appropriate support; the upper benchmark separates the latter from those students who are expected to cope without the provision of additional support. For 2010 many HE institutions have elected to use the NBT tests for placement purposes, together with the statutory NSC.

Understandably there has been some concern over the 2008 NSC results in mathematics. Many institutions are currently experiencing poorer than expected mid-year mathematics results. The challenge for HE is to make optimum use of the NSC and NBT results, in order to admit, place and support students in their mathematics studies.

This paper provides some background information on the assessment landscape out of which the NBTs developed and uses the results of a few universities to compare the mathematics performance of first year students in 2009. It aims to assist decision makers in HE in their interpretation of student results and the subsequent placement of students into various programmes.
Lie symmetry methods and wave equation on a manifold

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SAMS Subject Classification: Lie Groups and Transformations

We study and classify the conservation laws of the combined nonlinear KdV, Camassa-Holm, Hunter-Saxton and the inviscid Burgers equation which arises in, inter alia, shallow water equations. It is shown that these can be obtained by variational methods but the main focus of the presentation is the construction of the conservation laws as a consequence of the interplay between symmetry generators and ‘multipliers’, particularly, the higher-order ones.
On Wielandt near-rings

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SAMS Subject Classification: Algebra

Let $G$ be an additive group, and let $H$ be a subgroup of $G^k$ for some $k \in \mathbb{N}$. The set $M_0(G, H, k)$ of zero-preserving self-maps $a$ of $G$ such that $a(H) \subseteq H$ is a near-ring with respect to pointwise addition and composition of mappings, called a Wielandt near-ring. Primeness in Wielandt near-rings was studied by Veldsman [2]. Let $G$ be a $T_0$ (and hence completely regular) topological group. We consider the subnear-ring $N_0(G, H, k)$ of $M_0(G, H, k)$ which consists of continuous functions. Note that if $H = 0$ or $G^k$, then $N_0(G, H, k) = N_0(G)$, the near-ring of all zero-preserving self-maps of $G$. We will study primeness in $N_0(G, H, k)$, and will, inter alia, generalise some results of Veldsman [2] and Booth and Hall [1].

References

The problem to be investigated in this study will be the role and influence of the spinal ligaments on spinal stability. The importance of the spinal ligaments and the effect on the total spinal stability have not been emphasized sufficiently in the literature and is currently a big field of interest world wide.

The spinal column has both intrinsic and extrinsic stability:

Intrinsic stability results from the opposing forces of (a) ligaments restraining vertebral motion, and (b) pressure within the nucleus pulposus tending to push the vertebrae apart (Nixon and Brown, 1986:100).

Extrinsic stability results largely from trunk musculature and intra-abdominal pressure, which is in turn maintained by abdominal wall musculature (Nixon and Brown, 1986:100).

A study was done on the muscles acting on the L4/L5 joint of the lumbar spine (Potvin and Brown, 2005:973-980). Bergmark (1989) was the first to fully define and examine the mechanical stability of a muscular system which can be considered stable when the potential energy, $V$(a function of several variables), of the entire system is at a relative minimum. A stable system must always be able to return to its original state of equilibrium in response to perturbations around this original state.
Spectrally arbitrary, nonderogatory matrix factorization over a general field

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SAMS Subject Classification: Algebra

It is known (Sourour[4]) that a nonsingular, nonscalar matrix $A$, over a general field, may be factored as $A = BC$, in which the spectra of $B$ and $C$ are arbitrary, subject only to the obvious determinantal condition $\det A = \det B \det C$. Sourour then uses this result to develop a unified theory for a number of matrix factorization results, provided the underlying field has sufficiently many elements in terms of the order of the matrix being factored. In a recent development, Johnson and Zhang[3] shows that the factorization result of Sourour may be refined in the case of matrices over the complex field, by in addition requiring that the factors $B$ and $C$ be nonderogatory. The purpose of this presentation is to establish the validity of the latter result over a general field with at least four elements, and to use it to remove the requirement on the order of the underlying field in the unified theory of Sourour.

References

Spectral characterizations of scalars in a Banach algebra

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SAMS Subject Classification: Operator Algebra and Functional Analysis

For a complex Banach algebra $A$ with unit $1$, we give several characterizations of the scalars, that is, multiples of the identity. To a large extent, this work is a continuation and generalization of the work done on characterizations of the radical in Banach algebras. In particular it is shown that if $a \in A$ has the property that the number of elements in the spectrum of $ax$ is less than or equal to the number of elements in the spectrum of $x$ for all $x$ in an arbitrary neighbourhood of $1$, then $a$ is a scalar. Moreover, as a consequence of some of the results, new spectral characterizations of commutative Banach algebras are obtained. In particular, $A$ is commutative if and only if it has the property that the number of elements in the spectrum remains invariant under all permutations of three elements in some neighbourhood of the identity.
An abundance of Rado graphs

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SAMS Subject Classification: Combinatorics and Graph Theory

Richard Rado constructed in 1964 an (undirected) denumerable graph \( R \) with the natural numbers as vertex set and the following edges: For given \( m \) and \( n \) with \( m < n \), \( m \) is adjacent to \( n \) if \( n \) has a 1 in the \( m \)’th position of its binary expansion.

It is well known that \( R \) is a universal graph in the set \( \mathcal{I} \) of all countable graphs (since every graph in \( \mathcal{I} \) is an induced subgraph of \( R \)). From other known properties of \( R \) it can easily be deduced that there are edge-disjoint copies of \( R \) in the complete denumerable graph \( K_\infty \) (since \( R \) is self-complementary) and also that some proper subgraphs of \( R \) are isomorphic to \( R \) (since \( R \) is indestructible).

In this talk we offer a number of new constructions of graphs isomorphic to the Rado graph \( R \) (called clones of \( R \)) and use them to prove the existence of many pairwise edge-disjoint, many pairwise vertex-disjoint and an abundance of different copies of \( R \) in \( K_\infty \) and in \( R \).
Visualise electromagnetic distribution in a closed environment

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We are constantly exposed to electromagnetic radiation from a variety of sources. In designing devices that discharge an electromagnetic voltage it is important to know the radiation pattern and the effect of the electromagnetic radiation on the environment. This article discusses methods to determine the electromagnetic voltage and visually display the distribution in a closed environment as well as the cumulative effect of different radiation sources in a closed environment.
A graph-theoretic proof of the non-existence of self-orthogonal Latin squares of order 6

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SAMS Subject Classification: Combinatorics and Graph Theory

The non-existence of a pair of mutually orthogonal Latin squares of order six is a well-known result in the theory of combinatorial designs. It was conjectured by Euler in 1782 and was first proved by Tarry [4] in 1900 by means of an exhaustive enumeration of equivalence classes of Latin squares of order six. Various further proofs have since been given [1, 2, 3, 5], but these proofs generally require extensive prior subject knowledge in order to follow them, or are ‘blind’ proofs in the sense that most of the work is done by computer or by exhaustive enumeration. In this talk we present a graph-theoretic proof of a somewhat weaker result, namely the non-existence of self-orthogonal Latin squares of order six, by introducing the concept of a self-orthogonal Latin square graph. The advantage of this proof is that it is easily verifiable and accessible to discrete mathematicians not intimately familiar with the theory of combinatorial designs. The proof also does not require any significant prior knowledge of graph theory.

References

Lie symmetry analysis of the Asian option

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SAMS Subject Classification: Probability Theory, Mathematical Statistics and Financial Mathematics

Asian options incorporate the average stock price in the terminal payoff. Examination of the Asian option partial differential equation (PDE) has resulted in many equations of reduced order that can generally be mapped into each other, although this is not always shown. In the literature these reductions and mappings are obtained through the use of transformations that are typically acquired via inspection or ad hoc methods. We use Lie point symmetries with Lie’s systematic and algorithmic methods to show that one can obtain the same aforementioned reductions and mappings.
How to make a first-year service course more interactive using an on-line course management system.

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SAMS Subject Classification: History and Teaching of Mathematics

Moodle is an internet-based course management programme that has the potential to enrich the learning experience for students. In this session I will demonstrate how Moodle was used in a first-year Life Science Mathematics module, focusing on the set-up of multiple choice and type-the-answer quizzes. The students’ opinions on using Moodle will be presented, along with the challenges a new Moodle user may encounter and some suggestions on overcoming these.
Study and implementation of moving finite element method in 2D

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SAMS Subject Classification: Computational Methods

Nonlinear partial differential equations often lead to strongly non smooth (shock or near-shock) solutions. In that situation classical fixed grid methods fail to produce an adequate numerical approximation in a reasonable time. Till now a number of computational algorithms were developed to overcome this difficulty. All of them allow computational grid to move together with critical regions of the solution automatically. However, already in a one dimensional case there are a number of challenges related to reliability and efficiency of the proposed methods. In higher dimensions the situation becomes even more complicated.

In this talk we present theoretical and numerical analysis of these techniques in the case of two spatial dimensions. The first part deals with the study of finite element discretization on moving grids. The second part includes practical implementation and tests of the algorithms on a representative set of differential equations.

References


Some optimal codes admitting the simple group $L_3(4)$ as an automorphism group

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SAMS Subject Classification: Algebra

Using a combination of techniques described in [1, 2] and in [3] we determine non-trivial binary codes and designs from the 2-modular representations of $L_3(4)$.

References

Temperature effects in flowing polymeric liquids

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SAMS Subject Classification: Mathematical Physics; Computational Methods; Applications of Mathematics to the Sciences

The mathematical discussion on the constitutive modeling of non-isothermal effects in the flow of viscoelastic fluids is still underway. What is now beyond doubt is that temperature changes in such flowing polymeric systems should at the very minimum capture the effects of the three processes: conductive heat transfer effects, entropic effects due to stress work and energetic effects due to the changes in the polymer orientations. Secondary effects, say due radiation and chemical reactions can be included or neglected depending on the exact nature of the physical situation. The deformation/relaxation processes encountered in various physical situations necessitate an appropriate modeling of the energetic effects either via the local fast variable or via the macroscopic conformational tensor. We discuss these issues with reference to applied physical problems involving transient non-isothermal flow of polymeric liquids.
Correspondence and canonicity for distributive modal logic

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SAMS Subject Classification: Mathematical Logic and Foundations

Sahlqvist’s theorem [1] identifies a large, syntactically specified class of modal formulas which are canonical and also locally elementary on Kripke frames. Via the well known duality theory linking frames and Boolean algebras with operators (BAO’s), this result can be equivalently reformulated in the purely algebraic setting of canonical extensions. This perspective immediately suggests generalizations of Sahlqvist’s theorem along algebraic lines, e.g., to the cases of distributive [2] or arbitrary lattices with operators.

We give a brief overview of Sahlqvist theory for modal logic (with BAO-based semantics) and distributive modal logic (with distributive lattice-based semantics) and also discuss the appropriate duality with the relational semantics. We formulate a distributive modal logic analogue of Goranko and Vakarelov’s inductive formulas [3]. Next we present the newly developed algorithm ALBA [4] which effectively captures these results via a handful of syntactic transformation rules exploiting the residuation of (dual) operators and some other properties of perfect distributive lattices.

In conclusion we discuss some preliminary results on the generalization of this algorithm and syntactic classes to logics with semantics based upon arbitrary, possibly non-distributive lattices.

References

Bianchi derivative characterization of Lovelock polynomial

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SAMS Subject Classification: Mathematical Physics

It is well known that the trace of Bianchi derivative of Riemann curvature yields the second rank symmetric (Einstein) tensor with vanishing divergence which is exactly what one obtains on variation of Einstein Hilbert action, the trace of Riemann (scalar curvature). We identify the fourth rank tensor which is a homogeneous polynomial in Riemann and the trace of its Bianchi derivative gives the analogous divergence free tensor which follows from variation of Lovelock action. This is a new characterization of Lovelock polynomial.
Average distance in weighted graphs

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SAMS Subject Classification: Combinatorics and Graph Theory

Let $G$ be a given connected graph on $n$ vertices. Suppose $N$ facilities are located in the vertices of the graph. We consider the expected distance between two randomly chosen facilities. This is modelled by the following definition:

Let $G$ be a connected graph and let each vertex have weight $c(v)$. The average distance of $G$ with respect to $c$ is defined as

$$
\mu_c(G) = \left(\frac{N}{2}\right)^{-1} \sum_{\{u,v\} \subseteq V(G)} d_G(u, v),
$$

where $N = \sum_{v \in V(G)} c(v)$, and $d_G(u, v)$ is the distance in $G$ between $u$ and $v$, i.e., the length of a shortest $u - v$ path in $G$.

In this talk we consider bounds on $\mu_c(G)$ in terms of properties of $G$. We specifically consider the case when $G$ is a tree.
In The law of the iterated logarithm for random Kolmogorov, or chaotic, sequen-
ces, (1987) Vov’k showed that the law of the iterated logarithm is satisfied
by an algorithmically random sequence, by virtue of its high descriptional com-
plexity.

In this talk we will look at some related properties than can be deduced from
a sequence being algorithmically random.
Mathematica V7: Interactive maths for teaching and research

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SAMS Subject Classification: History and Teaching of Mathematics

Mathematica is a high level programming environment and was first developed by Wolfram Research in 1988. Today it is widely used in both academia and industry and with the recent introduction of Mathematica 7, more than five hundred new features have been added. Most notably some that unify language and interface concepts.

A live demonstration will explain how to use Mathematica’s dynamic interactivity for teaching and research applications. Wolfram’s demonstrations project can also be used as an educational resource - giving students access to thousands of interactive graphing and calculation applications.
On the stochastic 3D Navier-Stokes alpha model of fluids turbulence and their relation with the 3D stochastic Navier-Stokes equations

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SAMS Subject Classification: PDE’s and Dynamical Systems

In this task, we prove the existence of probabilistic weak solutions for the 3D stochastic Navier-Stokes alpha model. Moreover, we investigate the asymptotic behavior as alpha approaches zero.
“Let us remember that mathematics begins with questions, and only ends in definitions.”

“The Socratic didactician would refuse to introduce the geometrical objects by definitions, but wherever the didactic inversion prevails, deductivity starts with definitions. The Socratic didactician rejects such a procedure. How can you define a thing before you know what you have to define?” - Hans Freudenthal (1973) in Mathematics as an Educational Task, Dordrecht: D. Reidel, p. 416.

The construction of definitions (defining) is a mathematical activity of no less importance than other mathematical processes such as solving problems, making conjectures, generalizing, specializing, proving, etc. It is therefore perhaps a step in the right direction that the current South African curriculum (2005) specifically states in regard to Learning Outcome 3: Shape, Space and Measurement for Grade 10 that the learner should:

- investigate necessary and sufficient conditions to define various polygons (including the isosceles, equilateral and right-angled triangles, the trapezium, the isosceles trapezium, kite, parallelogram, rectangle, rhombus, square, etc.)

But teaching learners, or even students and prospective mathematics teachers at university, how to define geometrical objects is fraught with several difficulties. The purpose of this paper to provide some theoretical perspectives, as well as some research data on the problem. For example, a distinction will be made between two kinds of defining in mathematics, namely, descriptive and prescriptive defining. Closely associated with defining is the process of classification, and in relation to the quadrilaterals, establishing a hierarchical classification is particularly problematic for many learners. Moreover, in any such approach attention should be given, either implicitly or explicitly, to necessary and sufficient conditions, as students often mistakenly regard necessary conditions as sufficient conditions for defining concepts.
Generalizing a hexagon result via proof

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SAMS Subject Classification: Geometry; History and Teaching of Mathematics

In the majority of textbooks at high school and university, the purpose of proof in mathematics is still presented almost exclusively as that of verification; i.e. only as a means of obtaining certainty and to eliminate doubt. However, proving is not just about making sure. Particularly, given the very high level of conviction one can nowadays obtain through many different computer programs, proof may instead serve the purpose of a logical explanation of why a certain result is true (see De Villiers, 2003). Moreover, a proof can also sometimes serve as a tool for discovery since it often provides valuable insight into why a result is true, hence immediately enabling one to generalize or vary the result in different ways. The purpose of this paper is to give one example of a relatively recent problem I worked on that illustrates this “discovery” function very well. The investigation started with the following interesting result that was discovered with the aid of Sketchpad: If \( ABCDEF \) is a hexagon with opposite sides parallel (not necessarily equal), then the respective centroids \( G, H, I, J, K \) and \( L \) of triangles \( ABC, BCD, CDE, DEF, EFA \) and \( FAB \), form a hexagon with opposite sides both equal and parallel.
On the time discrete approximation of the Brinkman-Forchheimer equations

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SAMS Subject Classification: PDE's and Dynamical Systems, Computational Methods

In this work, we study the structural stability of the fully implicit Euler scheme for the Brinkman-Forchheimer equations. More precisely, we consider the time discretization scheme of the unsteady Brinkman-Forchheimer equations, and we prove that it is well posed. We also derive some error estimates of the discrete solution. Next, with the aid of the discrete Gronwall lemma, we show that the numerical solution depends continuously on the Brinkman and the Forchheimer coefficient.
Contracting maximal ideals in rings of continuous functions

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SAMS Subject Classification: Topology

A subspace $S$ of a Tychonoff space $X$ is said to be $C_1$-embedded (see [2]) if whenever $Z$ is a zero-set of $X$ and $W$ a zero-set of $S$ disjoint from $Z$, then $Z$ and $W$ are completely separated in $X$. We extend this notion to the pointfree context, and then, by observing that $S$ is $C_1$-embedded in $X$ iff the frame homomorphism $\mathcal{O}X \to \mathcal{O}S$, induced by the subspace inclusion $S \hookrightarrow X$, is a $W$-map in the sense of [1], we show that $S$ is $C_1$-embedded in $X$ iff the ring homomorphism $C(X) \to C(S)$, given by $f \mapsto f|_S$, contracts maximal ideals to maximal ideals.

References

Joinings of $W^*$- and $C^*$-dynamical systems

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SAMS Subject Classification: Operator Algebra and Functional Analysis

We study the notion of a joining of two $W^*$-dynamical systems, generalizing ideas from measure theoretic ergodic theory. In particular we consider necessary and sufficient conditions for ergodicity and weak mixing in terms of joinings.

Joinings of an arbitrary collection of $C^*$-dynamical systems are defined in terms of free products of $C^*$-algebras, as an analogue of joinings of classical dynamical systems. We show how multi-time correlation functions appearing in quantum statistical mechanics naturally fit into this joining framework.
Continuous symmetries of difference equations

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SAMS Subject Classification: Lie Groups and Transformations.

Given an ordinary or partial differential equation, one can apply Lie algebra techniques to analyze the problem. It is commonly known that the number of independent variables can be reduced after the symmetries of the equation are obtained. One can determine the optimal system of the equation in order to get a reduction of the independent variables. We can as well, using the method, obtain new solutions from known ones. This feature is quite interesting because some differential equations have apparently useless trivial solutions, but applying Lie symmetries on them, more interesting solutions are obtained. The question arises when it happens that our equation contains a discrete quantity. In other words, can the same steps be performed when we have a difference equation? How do we find symmetries of difference equations and how do we use them? We perform an analysis on the work done by some researchers in the field and apply their results to some examples.
Binary codes and partial permutation decoding sets from the Johnson graphs

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SAMS Subject Classification: Combinatorics and Graph Theory

The Johnson graph, denoted by $J(n,k)$, is the graph of which the vertex-set is the set of all $k$-subsets of $\Omega = \{1,2,\ldots,n\}$, and any two vertices $u$ and $v$ constitute an edge $[u,v]$ if and only if $|u \cap v| = k - 1$. In this talk the codes and their duals generated by the adjacency matrix of $J(n,k)$ will be described.

It will be shown that in each case, the code has a basis comprising minimum weight vectors. The same does not apply to the dual codes, since if both $n$ and $k$ are even, then the minimum weight vectors do not span the dual code.

The codes from $J(n,k)$ are also the codes of the $1-\left(\binom{n}{k}, k(n-k), n(n-k)\right)$ design $\mathcal{D}$ which has the vertices of $J(n,k)$ and the supports of the incidence vectors of its adjacency matrix as its point-set $\mathcal{P}$ and its block-set $\mathcal{B}$ respectively.

It is known that the automorphism group of $J(n,k)$ is $S_n$, and it is shown by an explicit argument that the automorphism group of the code is also $S_n$ for $k > 2$, except when $k$ is odd and $n$ is even, in which case it is $S_{\binom{n}{k}}$. 3-PD-sets are obtained for the code in the case that $k \geq 4$ and $n \geq 8$ are even and $n \geq 2k$, and in the case that $k \geq 6$ is even and $n > 2k$ is odd.
Trifactorized locally finite groups with min-$p$ for every prime $p$

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SAMS Subject Classification: Algebra

A group $G$ is trifactorized if $G = AB = AC = BC$ with three subgroups $A$, $B$, and $C$ of $G$. Some structural theorems about trifactorized locally finite groups with minimum condition on $p$-subgroups for every prime $p$ are proved. For instance, it is shown that $G$ is locally supersoluble (locally nilpotent) if $A$ and $B$ are locally nilpotent and $C$ is locally supersoluble (locally nilpotent).
Traceability of oriented graphs with fixed girth

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SAMS Subject Classification: Combinatorics and Graph Theory

An oriented graph is $k$-traceable if each of its subdigraphs of order $k$ is traceable. The Traceability Conjecture states that for $k \geq 2$ every $k$-traceable oriented graph of order at least $2k - 1$ is traceable. For each $g \geq 4$ we establish an upper bound (linear in $k$) on the order of strong $k$-traceable oriented graphs with girth at least $g$ and show that the Traceability Conjecture holds for strong oriented graphs with girth at least 6.
Report of the work on regular closed extensions

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SAMS Subject Classification: Topology; Algebra

This is a report of our work in connection with regular closed extensions of a regular topological space. On one hand, there is a well known counter example of a regular $T_1$ space which cannot be embedded densely into any regular closed $T_1$ space, and the Tychonoff spaces can always be embedded into a regular closed $T_1$ space, while on the other hand there seems to be a detailed paper due to Porter and Votaw providing an extension of a regular $T_1$ space into $OCER$ spaces (= open combinatorially embedded regular spaces) which subsumes the problem of regular closed extensions. However, the problem still seems to be unsolved in the following sense: what are the class of regular spaces for which there exists a dense extension into a regular closed spaces.

The purpose of our project is to address this on the category $Frm$ of frames; the project is still incomplete and we report some of the results obtained so far.
The zero square radicals

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SAMS Subject Classification: Algebra

In this talk we define zero square and weakly zero square ideals. We then construct the weakly zero square and the zero square radicals and characterize strongly zero square near-rings in terms of the strongly zero square radical.
Spectral analysis of a population model

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SAMS Subject Classification: PDE’s and Dynamical Systems

We study a closed one-sex population migrating between “states” which may be correspond to geographical regions, social status, physiological features or other classifications. We assume that the demographic processes (ageing, births and deaths) occur on a much slower time scale than migrations. The existence of different time scales in the model is accounted for by introducing a small parameter $\varepsilon > 0$ describing the relative strength of these processes; thus the system becomes singularly perturbed.

The main result of the work is to find an explicit expression of the dominant spectral value of the generator of the semigroup for the system.
Junction conditions with the generalized Vaidya metric

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SAMS Subject Classification: Mathematical Physics

We generate the junctions conditions at the stellar surface by matching the generalized Vaidya metric to the interior spacetime which is spherically symmetric and radiating. We establish that the radial pressure depends on the magnitude of the heat flux and the relativistic mass function. Some physical consequences of our result are briefly explored.
Homology and cohomology for semi-abelian categories

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SAMS Subject Classification: Algebra

We give a definition of the zeroth homology and zeroth cohomology (when it exists) as left and right adjoints of a pullback functor. We give two equivalent constructions of the zeroth homology as well as two constructions of the zeroth cohomology, the first more general than the second.
On good reductions of function fields revisited

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SAMS Subject Classification: Number Theory; Algebra

The aim of this talk is to discuss certain questions and results related to good reduction of function fields in a general setting. These results have recently been needed by specialists working in area of Field Arithmetic, prompting this investigation. The work builds on and refines that done by a number of authors, but here restricting to the case of regular/good reduction.
While standard finite element methods are effective in solving certain boundary value problems, there are conditions in which they generate highly unsatisfactory results, such as in the convection-dominated case of the convection-diffusion equation. The use of a discontinuous Galerkin finite element method is one way to avoid some of the problems caused by these conditions.

I will discuss aspects of the analysis of these methods as well as their implementation, with particular reference to the diffusion and convection-diffusion equations.
Conformally related Petrov type III spacetimes

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SAMS Subject Classification: Mathematical Physics

We generate new exact solutions to the Einstein field equations for a perfect fluid source by performing a conformal transformation on a known perfect fluid spacetime of type III in the Petrov classification scheme. The Allnut (1981) solution is the only known solution of type III and was obtained using the Newman-Penrose formalism. The simplicity of the solution ostensibly lends itself to the conformal transformation approach to generating new solutions. This approach is algebraically special as it constitutes restrictions on the Weyl conformal tensor which essentially invokes symmetries on the spacetime manifold. Solutions of the Einstein field equations are important in relativistic astrophysics. They provide a means to study holistically the evolution and stability of celestial phenomena.

A theorem due to Defrise-Carter (1975) asserts the following. Suppose that a manifold \((M, g)\) is neither conformally flat nor conformally related to a generalised plane wave. Then a Lie algebra of conformal Killing vectors on \(M\) with respect to \(g\) can be regarded as a Lie algebra of Killing vectors with regard to some metric on \(M\) conformally related to \(g\). Therefore if a spacetime admits the conformal group \(C_s\), then either it is conformally flat \((s = 15)\), conformally related to a generalised plane wave \((s \leq 7)\), or the metric \(\bar{g}_{ab} = e^{2U}g_{ab}\) where \(g_{ab}\) admits an \(s\)-dimensional \((s \leq 6)\) isometry group. We have previously considered Petrov Type D spacetimes within the above framework and have been successful in generating new exact solutions with the aid of Lie group analysis methods (Hansraj, et al 2006). Additionally we have succeeded in obtaining new exact solutions for perfect fluids that are conformal to a vacuum (Ricci flat) spacetime such as the non-conformally flat Schwarzschild exterior solution.

The next stage is to determine whether new perfect fluid solutions can be obtained from known perfect fluid solutions. That is solutions that are non-Ricci flat. This has been achieved with the type III solution of Allnut (1981). This perfect fluid metric has the form

\[
\begin{align*}
  ds^2 &= z^2 \left[ -\frac{dt^2}{1 + t^2} + f dx^2 + \frac{t^2 (1 + t^2)}{f} dy^2 \right] + dz^2
\end{align*}
\]

where \(f = t^{2\beta} (1 + t^2)^{1-\beta}\) and \(\beta\) is a constant. In the conformally related regime, the conformal factor is computed after several arduous calculations and is given by

\[
e^U = - \left( ze^{-\frac{1}{\sqrt{1-\beta}}} \arctan \frac{t}{\sqrt{1-\beta}} \left[ -k \arctan \frac{t}{\sqrt{1-\beta}} + C \right] + kz + C_1 + C_2 \ln z \right)^{-1}
\]
The metric $\tilde{ds}^2 = e^{2U} ds^2$ is a new solution to the Einstein field equations for a perfect fluid source. The solution is under investigation for physical plausibility. In particular, we wish to study the pressure and energy density profiles as well as the principle of causality. Additionally if the solution admits an equation of state it will possess the desirable qualitative features of a realistic stellar model.
Let $G = (V, E)$ be a graph. A set $S \subseteq V$ is a restrained dominating set if every vertex in $V - S$ is adjacent to a vertex in $S$ and to a vertex in $V - S$. The restrained domination number of $G$, denoted by $\gamma_r(G)$, is the smallest cardinality of a restrained dominating set of $G$. In this talk, we provide a survey of results concerning restrained domination in graphs.
Searching for identity à la Gelfand

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SAMS Subject Classification: Operator Algebra and Functional Analysis

Strong algebraic conclusions can often be drawn by restricting the growth rate of powers of an operator (hence analytic conditions). We consider some growth conditions, imposed on a bounded linear operator $T$ on a Banach space $X$, which force the operator $T$ to be the identity operator $I$. Among the many results in this area, we discuss those of Gelfand, Ritt, and Grobler and Huijsmans.
Strong transversals in hypergraphs and double total domination in graphs

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SAMS Subject Classification: Combinatorics and Graph Theory

Let $H$ be a 3-uniform hypergraph of order $n$ and size $m$ and let $T$ be a subset of vertices of $H$. The set $T$ is a strong transversal in $H$ if $T$ contains at least two vertices from every edge of $H$. The strong transversal number $\tau_s(H)$ of $H$ is the minimum size of a strong transversal in $H$. We show that $7\tau_s(H) \leq 4n + 2m$ and we characterize the hypergraphs that achieve equality in this bound. In particular, we show that the Fano plane is the only connected 3-uniform hypergraph $H$ of order $n \geq 6$ and size $m$ that achieves equality in this bound.

A set $S$ of vertices in a graph $G$ is a double total dominating set of $G$ if every vertex of $G$ is adjacent to at least two vertices in $S$. The minimum cardinality of a double total dominating set of $G$ is the double total domination number $\gamma_{t,2}(G)$ of $G$. Let $G$ be a connected graph of order $n$ with minimum degree at least three. Using our hypergraph results, we show that $\gamma_{t,2}(G) \leq 6n/7$ with equality if and only if $G$ is the Heawood graph.
Fredholm theory in Banach algebras

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SAMS Subject Classification: Operator Algebra and Functional Analysis

Fredholm operators were originally defined in an algebra of bounded linear operators on a Banach space. In [2], R.E. Harte used Atkinson’s theorem to generalise this definition to Fredholm elements in a Banach algebra relative to some fixed bounded Banach algebra homomorphism. Weyl and Browder elements were also introduced and results related to elements with these properties were obtained. Later the boundedness of the Banach algebra homomorphism was dropped from the definition and most results could still be obtained as H. du T. Mouton and H. Raubenheimer showed in [3], using a result of J.J. Grobler and H. Raubenheimer in [1]. In this talk the definitions of Fredholm, Weyl and Browder elements in a Banach algebra are given. Some inclusion properties of the sets of Fredholm, Weyl and Browder elements in relation to each other are shown, as well as some spectral inclusion properties. Most of these results in their original form are due to Harte. Finally, some properties of upper and lower semi-regularities due to V. Müller ([4]) are applied to some sets related to Fredholm Theory in Banach algebras.

References


On the stability criterion for an implicit evolution problem, based on a system of two nonlinear parabolic equations

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SAMS Subject Classification: PDE’s and Dynamical Systems

We construct an energy identity for an implicit evolution problem, which is based on a non-autonomous system of two nonlinear parabolic equations. From the energy identity, we construct a Lyapunov function and derive the conditions under which the null solution to the problem is stable or unstable. In other words, we use the direct Lyapunov stability/instability criterion for the null solution to the said implicit evolution problem. Problems of this nature occur either in heat transfer through surface radiation or permeable boundary Navier-stokes flows. The criterion thus derived will be tested on the two models describing the preceding phenomena. The existence of the ‘weak’ solution to this problem was confirmed in [1]. The computational confirmation of the solution to the problem appears in [2].

References
The semidirect product in a variety of right \( \Omega \)-loops

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SAMS Subject Classification: Algebra

Let \( C \) be a fixed variety of universal algebras which has among its operations a binary \( + \), a binary \( - \), and a nullary 0 satisfying the identities \( x + 0 = x \), \( 0 + x = x \), \( (x - y) + y = x \) and \( (x + y) - y = x \). In this talk we construct the semidirect product (see [2]) in \( C \) as the set-theoretical (cartesian) product equipped with the \( \Omega \)-algebra structure. This construction helps to develop the theory of crossed modules in \( C \) independently of the definition given in [3].

References

Lattice-valued categories of lattice-valued convergence spaces

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SAMS Subject Classification: Topology

In this talk we have another look at the category $SL$-$GCS$ of lattice-valued generalized convergence spaces [2, 3]. We show that extending the structure of continuous convergence (which makes $SL$-$GCS$ a cartesian closed category) from the set of continuous mappings between spaces to a set $F$ of arbitrary mappings between spaces, one of the axioms satisfied by the objects in $SL$-$GCS$ may no longer be valid for $F$. This poses the question: ”How far is $F$ away from being in $SL$-$GCS$?” Using a frame as lattice, this question can be answered if we attach ”grades of continuity” to the mappings in $F$. In this way, we are naturally led to the concept of a lattice-valued category in the sense of Šostak [4, 5, 6]. Such an $L$-category consists of an ordinary category [1] of ”potential objects” and ”potential morphisms” together with two $L$-classes, assigning a grade of being an object and of being a morphisms of the $L$-category. We describe initial constructions and function spaces of the resulting $L$-category of $L$-convergence spaces. Also we use Šostak’s concept of $L$-category and study ”how far away a lattice-valued convergence space is from being a lattice-valued topological space”.

References

Precategories, abstract radicals, and factorization systems

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SAMS Subject Classification: Algebra

We explain that certain analogy between the so-called Kurosh - Amitsur radicals and factorization systems of category theory leads to a mutual generalization that in turn suggests a new definition of a radical. This is joint work with László Márfi, partly published as [G. Janelidze and L. Márfi, A simplicial approach to factorization systems and Kurosh-Amitsur radicals, Journal of Pure Applied Algebra 213, 2009, 2229-2237].
We will recall basic properties and some examples of generalized central extensions defined via Huq commutators [2] (see also [1]) in semi-abelian categories. Those properties will be used to prove that the composites of central extensions determine a relative semi-abelian category structure [3] on the ground category. We will then briefly describe how the relative versions of basic homological lemmas [4,5] apply to this case.

References

Construction of maximal invariant subspaces for the classes of $H$-expansive matrices

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SAMS Subject Classification: Algebra

We considered indefinite inner products given by a square real invertible symmetric matrix $H = H^T : [x, y] = (Hx, y)$. On the Euclidean space equipped with this indefinite inner product, we consider matrices $A$ for which $A^*HA - H$ is nonnegative. Such matrices are called $H$-expansive matrices.

We are interested in the construction of complex (as well as real) $A$-invariant maximal $H$-nonnegative subspaces. The complex case has already been shown if one uses a suitable Cayley transform. The problem arises when $A$ is real and $A^THA - H$ is nonnegative and $A$ has both 1 and -1 as eigenvalues.
Bounds on the broadcast chromatic number for cubic graphs

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SAMS Subject Classification: Combinatorics and Graph Theory

Goddard, Hedetniemi, Harris and Rall introduced the broadcast chromatic number. They defined a broadcast coloring of order $k$ as a function from the vertex set $V$ to the set $\{1, \ldots, k\}$ such that equality between function values of $u$ and $v$ implies that the distance between $u$ and $v$ is more than the function value of $u$. The minimum order of a broadcast coloring is called the broadcast chromatic number of $G$, and is denoted by $\chi_b(G)$.

For this talk, we are considering bounds on the broadcast chromatic number for cubic graphs.
Zeros of the hypergeometric polynomials

\[ _2F_1(-n, -x; a; t) \]

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SAMS Subject Classification: Real and Complex Analysis

We examine the zeros of the hypergeometric polynomials

\[ _2F_1(-n, -x; a; t), \quad a, t \in \mathbb{R}, n \in \mathbb{N} \]

for various values of the parameters \( a \) and \( t \). We focus on Meixner polynomials that satisfy a discrete orthogonality relation for certain values of these parameters as well as on the behavior of the zeros of these polynomials as the parameter \( t \) tends to infinity.
Convergence of ray sequences of Padé approximants for a class of hypergeometric functions

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The denominator polynomial \( Q_{mn}(z) \) in the \([m/n]\) Padé approximant \( P_{mn}(z)/Q_{mn}(z) \) for \( 2F_1(a, 1; c; z) \) was explicitly evaluated by Padé for \( m \geq n - 1 \) and \( c \notin \mathbb{Z}^- \). We show that for \( c > a > 0 \) and \( m \geq n - 1 \), the poles of \( P_{mn}(z)/Q_{mn}(z) \) lie on the cut \((1, \infty)\). We deduce that the sequence of approximants \( P_{mn}(z)/Q_{mn}(z) \) converges to \( 2F_1(a, 1; c; z) \) as \( m \to \infty \), \( n/m \to \rho \) with \( 0 < \rho \leq 1 \), uniformly on compact subsets of the unit disc \( |z| < 1 \) for \( c > a > 0 \).
A complete analysis of the conservation laws of the family of nonlinear equations

$$\alpha(u_t + 3uu_x) + \beta(u_{txx} + 2u_xu_{xx} + uu_{xxx}) - \gamma u_{xxx} = 0.$$  

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SAMS Subject Classification: Lie Groups and Transformations  

We study and classify the conservation laws of the combined nonlinear KdV, Camassa-Holm, Hunter-Saxton and the inviscid Burgers equation which arises in, inter alia, shallow water equations. It is shown that these can be obtained by variational methods but the main focus of the presentation is the construction of the conservation laws as a consequence of the interplay between symmetry generators and ‘multipliers’, particularly, the higher-order ones.
Lagrangian approach to a coupled Lane-Emden system

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SAMS Subject Classification: ODE’s and Dynamical Systems

Systems of Lane-Emden equations arise in the modelling of several physical phenomena, such as pattern formation, population evolution and chemical reactions. In this talk we construct Noether and partial Noether operators corresponding to a Lagrangian and a partial Lagrangian for a coupled Lane-Emden system. The first integrals with respect to Noether and partial Noether operators are obtained for the Lane-Emden system under consideration. We show that the first integrals for both the Noether and partial Noether operators are the same. However, the gauge function is different in certain cases.
Enumeration of self-orthogonal Latin squares

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The enumeration of self-orthogonal Latin squares (SOLS) of a given order seems to be an open problem in the literature on combinatorial designs. The existence of at least one SOLS is guaranteed for any order except 2, 3 and 6, but it is not known how many of these squares of a given order exist. In this talk we present enumeration tables of unequal SOLS, idempotent SOLS, isomorphism classes of SOLS and isotopy classes of SOLS. The isotopy classes are enumerated by an (almost) exhaustive computerised tree search which generates a representative from each isotopy class, whereafter the automorphism groups of these representatives are used together with results from abstract algebra in order to enumerate unequal SOLS, idempotent SOLS and isomorphism classes of SOLS. Finally, the results are validated by using an alternative computerised tree search method for all four classes of SOLS. The results appear as sequences A160365, A160366, A160367 and A160368 in Sloane’s Online Encyclopedia of Integer Sequences [1].

References

Risk measures via vector lattice theory

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SAMS Subject Classification: Probability Theory, Mathematical Statistics and Financial Mathematics

A risk measure $\rho$ maps random variables, $X$, to real numbers. We consider $\rho(x)$ to be the minimum extra cash (capital) added to $X$ to make the position $X$ financial acceptable. Risk measures are considered in the light of the notions of coherency and convexity.
Asymptotic analysis of singularly perturbed dynamical systems: Numerical experiments

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SAMS Subject Classification: Computational Methods; Applications of Mathematics to the Sciences

In this talk we consider the approximation of kinetic equations using the Chapman-Enskog procedure. The results of computational experiments are presented for a model of age-structured population of soles, showing effects of the different kind of correctors applied for the accuracy of the approximation.
A finite-difference method for the valuation of variance swaps in Barndorff-Nielsen-Shephard stochastic volatility model

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SAMS Subject Classification: Probability Theory, Mathematical Statistics and Financial Mathematics

We develop a finite-difference method for valuing variance swaps in a Barndorff-Nielsen and Shepherd (BNS) stochastic volatility model. This approach was used by Little and Pant [5] in a diffusion model. We therefore generalize their results.

References

Maps on noncommutative Orlicz spaces

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SAMS Subject Classification: Operator Algebra and Functional Analysis; Mathematical Physics

We extend the construction of noncommutative Banach Function Spaces pioneered by Dodds, Dodds, de Pagter, et al, and use this generalisation to establish a noncommutative version of the Pistone-Sempi argument in the context of semifinite von Neumann algebras. We then pass to the question of lifting important classes of positive maps defined on von Neumann algebras, to maps on the corresponding noncommutative Orlicz spaces. We focus in particular on describing noncommutative analogues of composition operators.
The use of a computer algebra system to illustrate input and output waves for the telegraph equation

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SAMS Subject Classification: Applications of Mathematics to the Sciences

It would be difficult to imagine a world without communication systems. In order to optimise communication systems, it is necessary to determine possible power and signal losses in the system, since all systems have such losses. In this paper a mathematical derivation for the telegraph equation in terms of voltage and current for a section of a transmission line is investigated. The formulae for voltage and current involved in the telegraphic equations are not explicitly and analytically derived in literature for engineers. Also input or output waves for the telegraph equations are not usually given in the literature. This leaves a theoretical and visual gap seldom crossed by students in Electrical Engineering. This is of great concern since most of these students will be working as technicians and they will not really know what they have calculated and what the input and/or output waves look like.

The main aim of this paper is to address this theoretical gap by deriving from basic principles the equations for telegraphic transmission in a guided system and by making use of a CAS to illustrate the shape of the input and/or output waves. Students do all these calculations in either mathematics or engineering but do not know what the signals look like.
Controlling a stopped diffusion process to reach a goal

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SAMS Subject Classification: Probability Theory, Mathematical Statistics and Financial Mathematics

We consider a problem of optimally controlling a two-dimensional diffusion process

\[
\begin{aligned}
dx_t^{\mu,\beta} &= \mu(x_t)dt + \beta(x_t)dB^1_t; \quad x(0) = x, \\
dy_t &= \alpha y_t dt + (\sigma \sqrt{x_t} + \gamma) y_t dB^2_t; \quad y(0) = y,
\end{aligned}
\]

initially starting in the interior of a domain \( D_\varphi = \{(x, y) \in \mathbb{R}_+^2 : \varphi(x) < y < \theta\varphi(x)\} \) until it reaches the line \( y = \theta\varphi(x) \) at a stopping time \( \tau \leq T_0 \), where \( T_0, \alpha, \sigma, \gamma > 0 \) and \( \theta > 1 \) are fixed positive constants and \( \varphi(x) \) is a given positive strictly increasing, twice continuously differentiable function on \((0, \infty)\). The goal is to maximize the probability criterion

\[
\sup_{(\mu, \beta) \in \mathcal{M}(x)} \mathbb{P}\{y_\tau = \theta\varphi(x^{\mu,\beta}_\tau), \quad \tau \leq T_0 | x(0) = x, y(0) = y\}, \quad x, y \in D_\varphi
\]

over a class of admissible controls \( \mathcal{M}(x) \) consisting of bounded, Borel measurable functions. Under suitable conditions, it is shown that the maximal probability is given explicitly and the optimal process is determined explicitly by

\[
\rho(\varphi(x), \varphi'(x), \varphi''(x)) = \sup \left\{ \frac{\mu(x)\varphi'(x) - (\alpha - \frac{1}{2}(\sigma \sqrt{x} + \gamma)^2)\varphi(x)}{\beta(x)^2} : (\mu, \beta) \in \mathcal{M}(x) \right\}.
\]
Analysis of MHD mixed convection about a vertical plate embedded in porous medium

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SAMS Subject Classification: Computational Methods; Applications of Mathematics to the Sciences

Mixed convection flow with simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors and underground energy transport. In this paper, mixed convective boundary layer flow past a vertical porous plate embedded in a saturated porous medium with a constant heat flux and mass transfer in the presence of a magnetic field is investigated. Using the Boussinesq and boundary-layer approximations, the fluid equations for momentum, energy balance and concentration governing the problem are formulated. These equations are solved numerically by using the most effective Newton-Raphson shooting method along with fourth-order Runge-Kutta integration algorithm. The results reveal among other things that for positive values of the buoyancy parameters, the skin friction increased with increasing values of Eckert number (Ec) and magnetic field intensity parameter (M) and decreased with increasing values of Schmidt number (Sc) and permeability parameter (K).
Centralizers of matrices

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SAMS Subject Classification: Algebra

Since the minimum polynomial and characteristic polynomial of a non-scalar 2×2 matrix in $M_2(F)$, the full 2×2 matrix ring over the field $F$, coincide, the centralizer in $M_2(F)$ of a 2×2 matrix is known from the more general result, Theorem 5 on page 23, in [2]. As a corollary of this result a concrete description of the centralizer of a 2×2 matrix over an integral domain $D$ in $M_2(D)$ can be derived.

In this talk the concept of a $k$-matrix in $M_2(R/\langle k \rangle)$, where $R$ is an arbitrary unique factorization domain (UFD) and $k$ an arbitrary nonzero nonunit in $R$, is introduced. We obtain a concrete description of the centralizer of a $k$-matrix $\hat{B}$ in $M_2(R/\langle k \rangle)$ as the sum of two subrings $S_1$ and $S_2$ of $M_2(R/\langle k \rangle)$, where $S_1$ is the image (under the induced epimorphism from $M_2(R)$ to $M_2(R/\langle k \rangle)$) of the centralizer in $M_2(R)$ of a pre-image of $\hat{B}$, and where the entries in $S_2$ are intersections of certain annihilators of elements arising from the entries of $\hat{B}$. It turns out that if $R$ is a principal ideal domain (PID) then every matrix in $M_2(R/\langle k \rangle)$ is a $k$-matrix for every $k$. However, this is not the case for UFD's in general. Moreover, for every factor ring $R/\langle k \rangle$ with zero divisors and every $n \geq 3$ there is a matrix for which the above mentioned description is not valid.

References

The versatile Ruder Josip Bošković

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SAMS Subject Classification: History and Teaching of Mathematics

The International Year of Astronomy (IYA2009) is a year-long celebration of astronomy, taking place in 2009 to coincide with the 400th anniversary of the first recorded astronomical observations with a telescope by Galileo Galilei and the publication of Johannes Kepler’s Astronomia nova in the year 1609.

This talk deals with the life of the Jesuit priest, poet and diplomat Ruder Josip Bošković (born in 1711 in Ragusa, today Dubrovnik in Croatia, died in 1787 in Milan, Italy), and his contributions to astronomy, mathematics and physics.
We present a mathematical model of the cerebrospinal fluid (CSF) formation based on fluid mechanics concepts. It was assumed that CSF formation begins as plasma, and filtered across permeable boundaries of choroidal capillaries. We use the non-linear Stokes Equation with permeable boundary conditions to model this formation. The cerebral blood capillary is modeled as a micro vessel and blood flow in a micro vessel as a two-phase flow: the deformable blood cells phase and the liquid plasma phase. The principles and methods of “effective area” for studying the penetration of fluid into permeable walls, was used to investigate the filtrate momentum flux from the intracranial capillary wall, through the pia matter and epithelia layer of the choroid plexus, into the subarachnoid space. The existence of a unique weak solution is proved by using the weak formulation, the Riesz-representation Theorem and an application of Ladyzhenskaya’s work on Viscous Incompressible Flows.
Effect of road block on traffic flow

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SAMS Subject Classification: PDE’s and Dynamical Systems

A road with two lanes in the same direction with no off-ramps or on-ramps is considered. The model is used in which the traffic flux depends only on the traffic density. The traffic density satisfies a first order quasi-linear partial differential equation in conserved form. A road block is set up at time $t = 0$ which reduces the two lanes to one lane. When the traffic density reaches a critical value which depends on the ratio of the speed limit in the open road to the speed limit in the road block a shock forms at the entrance to the road block. The shock travels backwards from the road block into the oncoming traffic and is such that the traffic flow through the road block is maximised. The road block is removed after a time $T$. Using the properties of characteristic curves the maximum length of the tailback in light traffic after the roadblock is removed is obtained. For heavy traffic the tailback continues to grow indefinitely and the effect of the road block never clears.
Application of symmetry analysis to a model for sexually transmitted diseases

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SAMS Subject Classification: Lie Groups and Transformations

Lie group analysis is applied to a Mathematical model which describes sexually transmitted diseases formulated by Hadeler and Castillo-Chavez. Several instances of integrability even linearity are found which lead to the general solution of the model. A discussion of such solutions is presented and it is shown how they complement Hadeler and Castillo-Chavez’s qualitative analysis.
Notes concerning characterizations of quasi $F$-frames

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SAMS Subject Classification: Topology

In this talk, we give several characterizations of quasi $F$-frames. To name a few, let $RL$ denote the ring of real-valued continuous functions on a completely regular frame $L$. Then, $L$ is quasi-$F$ iff $\text{Ann}^2(\alpha) + \text{Ann}^2(\beta) = RL$ whenever $\alpha + \beta$ is not a zero-divisor. A commutative ring is said to be quasi-Bézout if every finitely generated ideal that contains a non-zero divisor is principal. We show that $L$ is quasi-$F$ iff $RL$ is quasi-Bézout. Further, $L$ is quasi-$F$ iff $\beta L$ is quasi-$F$, iff $\lambda L$ (its universal Lindelöfication) is quasi-$F$, iff $\upsilon L$ (its Hewitt real compactification) is quasi-$F$. 
A mathematical model of mixed infection in Tuberculosis (TB)

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SAMS Subject Classification: ODE’s and Dynamical Systems

We use ordinary differential equations to develop a model of mixed infection in TB. It is analysed mathematically by finding its equilibrium points and their stability. The model is shown to have multiple endemic equilibria and also exhibits the phenomenon of backward bifurcation whenever its basic reproductive number, $R_0$, is less than unity for certain parameter values. Numerical analysis of the model shows that mixed infection may help explain high TB incidence in certain areas.
Interlacing of the zeros of Jacobi polynomials with different parameters

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SAMS Subject Classification: Real and Complex Analysis

We prove results for the interlacing of zeros of Jacobi polynomials of the same or adjacent degree as one or both of the parameters are shifted continuously within a certain range. Numerical examples are given to illustrate situations where interlacing fails to occur.
Stability of viscoelastic flows

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SAMS Subject Classification: PDE’s and Dynamical Systems; Mathematical Physics

We consider a model of a viscoelastic fluid in which the classical Oldroyd-B model is extended to account for shear-dependent viscosity. The formulation was proposed in [1] to model the flow of blood. Suitable forms of the energy are derived and sufficient conditions for stability obtained. These conditions depend on relationships between a nondimensional parameter, the Weissenberg number, and the maximum rate of change of the viscosity. The conditions are illustrated by considering a simple model problem.

References

Exact solutions of the two-dimensional fin problem with temperature dependent thermal conductivity and heat transfer coefficient

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SAMS Subject Classification: Lie Groups and Transformations; PDE’s and Dynamical Systems; Mathematical Physics

This study investigates solutions of two-dimensional nonlinear fin problem with both thermal conductivity and heat transfer coefficient being given as power laws of temperature. The nonlinear problem considered here is linearized by a point transformation when the exponents of the power laws for heat transfer coefficient and thermal conductivity are equal. One dimensional optimal system of subalgebras is constructed for the point symmetries admitted by the governing equation with different exponents of the power laws and reductions are performed. Exact solutions satisfying the realistic boundary conditions are constructed. The effect of the physical parameters such as aspect ratio, thermo-geometric fin parameter, heat flux and fin efficiency are analyzed.
Group classification of the general second-order system of diffusion equations

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SAMS Subject Classification: Lie groups and Transformations

The purpose of this work is to perform group classification of a coupled system of general second-order diffusion equations. The functional forms of the arbitrary elements in the system are specified via the classification with respect to low dimensional Lie algebras.
Repairing description logic ontologies

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SAMS Subject Classification: Mathematical Aspects of Computer Science

An ontology is a formal representation of the taxonomic structure of a domain of interest. Ontologies are modelled by identifying relevant concepts (classes of individuals) present in a domain and applicable relationships that may exist among these concepts. These classes, individuals and relationships may be combined to form axioms (formulas) which describe the interactions among concepts in the domain. A finite set of such axioms, which we call a Knowledge-base (KB), defines an ontology.

For instance an ontology \( O = \{ 'Car is a Vehicle', 'Sedan is a Car' \} \) formalizes a car domain. It is worth noting that an implicit conclusion from \( O \) is ‘Sedan is a Vehicle’ drawn from the premises: ‘Sedan is a Car’ and ‘Car is a Vehicle’. This ability to derive implicit information is easier to automate when the axioms are expressed in a logical formalism rather than natural language.

Description Logics (DLs) are a group of logical formalisms that may be used to define ontologies. They also happen to be syntactic variants of L2, the two-variable fragment of first-order logic. Inference engines called DL reasoners are used to perform automated reasoning over ontologies and they provide a service of implicit knowledge extraction, which gives ontologies many applications in computer systems.

The process of building ontologies has been greatly simplified with the advent of graphical ontology editors such as SWOOP, Protege and OntoEdit. As a result of this there are a growing number of beginner ontology engineers attempting to build and develop ontologies. Because many of these people have very little experience with DL representation they are more inclined to make errors in the ontology development process. As such there is a need to extend current ontology editors with support modules to aid these ontology engineers in correctly designing and debugging their ontologies. This debugging procedure is called ontology repair.

In this work we propose an approach to ontology repair given a list of unwanted axioms, \( U \), in the ontology. We describe a method to efficiently remove these unwanted axioms (entailments) by analyzing their properties (in particular their justifications). A justification is the smallest (w.r.t. set inclusion) subset of a KB from which an entailment logically follows. If we gather all justifications for all axioms in \( U \) and place them in a set \( J \) then a root justification in \( J \) is one which has no proper subset also in \( J \).

A root entailment is an axiom \( \alpha \in U \) which has only root justifications in \( J \). A property of root entailment states that if one repairs \( \alpha \) first then this would cause other entailment(s) in \( U \) which have a dependency on \( \alpha \) to be
automatically repaired as well [1]. Therefore, before repair is executed, our intermediate goal is to pinpoint all root entailments in \( U \) without having to compute every justification in \( J \). This would make ontology repair more efficient because the number of modifications to the original ontology in order to reverse the entailments in \( U \) would be reduced.

References

Designs from maximal subgroups and conjugacy classes of finite simple groups

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SAMS Subject Classification: Algebra

Let $G$ be a finite simple group, $M$ be a maximal subgroup of $G$ and $C_g$ be the conjugacy class of $G$ containing $g$. In this talk we outline a new method for constructing $1-(v, k, \lambda)$ designs $D = (\mathcal{P}, \mathcal{B})$, where $\mathcal{P} = C_g$ and $\mathcal{B}$ is the set of all conjugates of $M$. The parameters $v$, $k$, $\lambda$ and further properties of $D$ are determined.
On the eccentric connectivity index of a graph

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SAMS Subject Classification: Combinatorics and Graph Theory

If $G$ is a connected graph with vertex set $V$, then the eccentric connectivity of $G$, $\xi^C(G)$, is defined as $\sum_{v \in V} \deg(v) \text{ec}(v)$ where $\deg(v)$ is the degree of a vertex $v$ and $\text{ec}(v)$ is its eccentricity. We obtain an exact lower bound on $\xi^C(G)$ in terms of order, and show that this bound is satisfied by the star graph. An asymptotically sharp upper bound is also derived. In addition, for trees of given order, when the diameter is also prescribed, precise upper and lower bounds are provided.
MTL-chains with modalities

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SAMS Subject Classification: Mathematical Logic and Foundations; Algebra

We discuss modal extensions of the logic MTL, or monoidal t-norm logic, which is a many-valued logic that may be considered the logic of ‘left-continuous t-norms’. A class of algebraic models for MTL consists of all MTL-chains, which are bounded chains in the lattice-sense, together with a t-norm operation $\circ$ and associated residual operation $\rightarrow$.

By adding modal operators to MTL we wish to increase its expressive power. Natural examples of such modal operators are the linear logic exponential $!$ and the Baaz delta. Algebraic models for modal MTL logics are then modal MTL-chains which are MTL-chains with an additional operation. We are interested in completions of modal MTL-chains and the equations that are preserved by such completions. In particular, we seek a Sahlqvist-like theory for modal MTL-chains analogous to the Sahlqvist theory for modal algebras, i.e., a syntactic descriptions of equations that are preserved by completion. As a special case, we consider the logic $LK^r$, which is essentially MTL with a linear logic modality, as defined in [2]. We also consider negative modalities such as $\neg$, which are order-reversing and are natural forms of negation.

References

Results on spectral continuity in ordered Banach algebras

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SAMS Subject Classification: Operator Algebras and Functional Analysis

Since J. D. Newburgh initiated the subject of spectral continuity in Banach algebras in 1951, it has been studied extensively. However, spectral continuity in the context of an ordered Banach algebra (OBA) has only been studied in recent years. Newburgh showed that the spectrum and spectral radius functions are upper semicontinuous. However, a well-known example by S. Kakutani illustrates that these functions are not continuous. In fact, this example shows that, in the case of an OBA, the spectrum and spectral radius functions are not even continuous on the algebra cone.

In this talk we will exhibit a number of theorems illustrating spectral continuity properties of positive elements. Among other things, we show (under natural conditions) that if $a$ is a positive element, then the restriction of the spectral radius function to the set $C(a) = \{x : a \leq x \text{ and } (ax \leq xa \text{ or } xa \leq ax)\}$ is continuous at $a$ (see [1]).

Given an element $a$ of a Banach algebra $A$, the boundary spectrum of $a$, denoted by $S_0(a)$, was defined in [2] as the set of all $\lambda \in \mathbb{C}$ such that $\lambda - a$ is an element of the boundary of the set of all non-invertible elements of $A$. Using this concept, we show that if $a$ is a positive element (relative to a closed and normal algebra cone $C$ in an OBA) such that $S_0(a) \cap \mathbb{R}^+$ consists of the spectral radius of $a$ only, then the restriction of the spectral radius function to $C$ is continuous at $a$ (see [3]).

We also consider some applications.

References

On path-width of a graph vs bridge number of a knot

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SAMS Subject Classification: Combinatorics and Graph Theory

We compare the path-width of a planar graph and the bridge number of the corresponding link. Is there a relationship between these two numbers? This study is an attempt to find a graph theoretical method of finding the bridge number of a link.
Solutions of the Einstein field equations with heat flow

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SAMS Subject Classification: Mathematical Physics

We study a shear-free spherically symmetric cosmological model with heat flow. The pivotal equation is given by:

\[ V D_{uu} + 2D_u V_u - D V_{uu} = 0, \]

where \( V \) and \( D \) are metric functions. A method of generating solutions has been developed by Deng (1989) who found solutions to this equation when simple forms of \( V \) or \( D \) are chosen. We show a new approach to solve this equation. Significantly it is the first time that an explicit relationship between \( V \) and \( D \) is provided. This method also recovers Deng’s solutions.
On diameter and inverse degree of a graph

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SAMS Subject Classification: Combinatorics and Graph Theory

The inverse degree $r(G)$ of a finite graph $G = (V, E)$ is defined as $r(G) = \sum_{v \in V} \frac{1}{\deg v}$, where $\deg v$ is the degree of vertex $v$. We establish inequalities concerning the sum of the diameter and the inverse degree of a graph which for the most part are tight. We also find upper bounds on the diameter of a graph in terms of its inverse degree for several important classes of graphs.
The $M_3$ versus $M_1$ problem

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SAMS Subject Classification: Topology

In this talk we state and discuss the forty eight year old problem, currently known as the $M_3$ versus $M_1$ problem. Some related thoughts in the setting of ordered topological spaces will be presented.
First integrals for systems via complex partial Lagrangians

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SAMS Subject Classification: Lie Groups and Transformations

The Noether operators and Euler-Lagrange equations are developed for a system of $m$ second-order ordinary differential equations (ODEs) with $m$ dependent variables in the complex domain with the help of complex Lagrangians. The system of $m$ ODEs in the complex domain can be split into $2m$ coupled real partial differential equations (PDEs) along with the constraint of the $2m$ Cauchy-Riemann (CR) equations. Thus a system of $4m$ PDEs for $2m$ real functions of two real variables is obtained. The complex Lagrangian splits into two real Lagrangians for a system of $4m$ PDEs which satisfy Euler-Lagrange equations in the real domain. Each complex Noether operator yields two real Noether operators of the real Lagrangians. The complex first integrals result in two real first integrals for the system of $4m$ PDEs obtained after splitting the system of $m$ ODEs in the real domain.
The Noether conservation laws corresponding to some Riemannian manifolds

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SAMS Subject Classification: Lie Groups and Transformations

We show that a large amount information can be extracted from a knowledge of the vector fields that leave the action integral invariant, viz., Noether symmetries. In addition to a larger class of conservation laws than those given by the isometries or Killing vectors, we may conclude what the isometries are and that these form a Lie subalgebra of the Noether symmetry algebra. We perform our analysis on versions of the Vaidiya metric yielding some previously unknown information regarding the corresponding manifold. Lastly, with particular reference to this metric, we show that the only variations on $m(u)$ that occur are $m = 0$, $m =$constant, $m = u$ and $m = m(u)$. 
Continuous symmetries of difference equations

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SAMS Subject Classification: Lie Groups and Transformations.

Given an ordinary or partial differential equation, one can apply Lie algebra techniques to analyze the problem. It is commonly known that the number of independent variables can be reduced after the symmetries of the equation are obtained. One can determine the optimal system of the equation in order to get a reduction of the independent variables. We can as well, using the method, obtain new solutions from known ones. This feature is quite interesting because some differential equations have apparently useless trivial solutions, but applying Lie symmetries on them, more interesting solutions are obtained. The question arises when it happens that our equation contains a discrete quantity. In other words, can the same steps be performed when we have a difference equation? How do we find symmetries of difference equations and how do we use them? We perform an analysis on the work done by some researchers in the field and apply their results to some examples.
The Noether point symmetries of a Lane-Emden-type equation

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SAMS Subject Classification: ODE’s and Dynamical Systems

We classify the Lane-Emden-type equation \( xy'' + ny' + x^n f(y) = 0 \) with respect to the standard Lagrangian \( L = \frac{1}{2}x^n y'^2 - x^n + (v-1) \int f(y) \, dy \) according to the Noether point symmetries it admits. First integrals of the various cases, which admit Noether point symmetries, and reduction to quadratures for these cases are obtained. Six cases result in new solutions.
A dynamic model of substance abuse: the case of ‘meth’

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SAMS Subject Classification: ODE’s and Dynamical Systems; Applications of Mathematics to the Sciences

The global rise in the use of methamphetamine has been documented to have reached epidemic proportions. Researchers have focussed on the social implications of the epidemic. A typical drug use cycle consists of concealed drugs use after initiation, addiction, treatment-recovery-relapse cycle, whose dynamics are not well understood. The model by White and Comiskey, on heroin epidemics, treatment and ODE modelling, is modified to model the dynamics of methamphetamine use in a South African province. The analysis of the model is presented in terms of the methamphetamine epidemic threshold $R_0$. It is shown that the model has multiple equilibria and using the center manifold theory, the model exhibits the phenomenon of backward bifurcation where a stable drug free equilibrium co-exists with a stable drug persistent equilibrium for a certain defined range of $R_0$. The stabilities of the model equilibria are carried out and persistence conditions established. Furthermore, numerical simulations are done including fitting the model to the available data on the number of patients with methamphetamine problems. The implications of the results to drug policy, treatment and prevention are discussed.
Characterization of stratified $L$-topological spaces by convergence of stratified $L$-filters

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SAMS Subject Classification: Topology

$L$-sets over a base set $X$ are generalizations of classical sets where subsets are not specified by characteristic functions from $X$ to $\{0,1\}$ but rather by functions from $X$ to a lattice $L$. For an $L$-set $a \in L^X$ and an element $x \in X$, $a(x)$ is interpreted as the grade of membership of $x$ in $a$. Stratified $L$-topological spaces are generalizations of topological spaces to the $L$-set case [1]. In [2], stratified $L$-generalized convergence spaces (analogous to classical convergence spaces) are defined, with the underlying lattice $(L, \leq, \wedge)$ being a frame. The resulting category $\text{SL-GCS}$ is topological over $\text{Set}$ and is Cartesian-closed [2]. $\text{SL-TOP}$, the category of stratified $L$-topological spaces, is isomorphic to a reflective subcategory of $\text{SL-GCS}$ [2]. In [3] various subcategories of $\text{SL-GCS}$ are investigated. The results of [2] and [3] are now extended to more general enriched lattices $(L, \leq, *, \otimes)$. Finally axiom schemes for $L$-topological spaces based on $L$-filters (which lead to isomorphic categories in the frame case [4]) are investigated in the more general case and conditions for isomorphism between their categories are explored.

References

Sex-structured population models

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SAMS Subject Classification: PDE’s and Dynamical Systems.

Populations are often sexually dimorphic. An extreme example of sexual dimorphism is found in the genus Osedax of polychaete worms, which lives on whale falls. The females feed on the bones of the dead whale; the males live inside the females and do not develop past their larval stage, except to produce large amounts of sperm. In the majority of scale insects, females are highly modified (eyeless and wingless, with non-functional appendages and reduced segmentation), attached permanently to their host plants, while males are rather ordinary though delicate insects, smaller and winged. In this talk we highlight some major differences. We discuss two-sex models that assume no added structure and thereafter turn to age-structured two-sex models.
Analytic aspects for fluid-structure interaction and numerical simulations for structure dynamics and fluid flow, test examples

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SAMS Subject Classification: Mathematical Physics

It is observed that when a fluid interacts with a solid structure, exerting pressure, that may cause deformation in the structure and, thus, alter the flow of the fluid itself. This deformation of a solid structure, changes the boundary condition of the fluid problem. These problems appear to have wide engineering applications such as the flow of gases in internal combustion engines where the piston head has a periodic motion with respect to the cylinder walls, modifying the available flow volume accordingly.

In the current work we present some analytic aspects related to fluid structure coupling and in order to test some numeric schemes, for structure and for fluid flow separately, we present some numerical simulations.

References

Consistency and convergence of SPH approximations

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SAMS Subject Classification: Computational Methods

Smoothed particle hydrodynamics, or SPH, is a widely used numerical technique for solving differential equations. There is, as yet, no complete analysis in the literature which shows under which conditions the method is consistent and converges. We aim to contribute towards such an analysis by considering two aspects of SPH: (1) the integral formulations used to approximate functions and their derivatives, and (2) the discretisation technique used to approximate these integrals numerically. With regard to (1) we explain the conditions placed on kernels in the literature, and also show how simpler, linear kernels can be used. We also present an as yet unpublished but essential condition on the continuity at the boundary of the problem domain. Our work in (2) is not yet complete but from our tests we show a sensitivity to the distribution of discretisation points. We are looking at several quadrature techniques and performing various analyses to try to both explain and mitigate these issues.
New exact models in radiating stellar collapse

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SAMS Subject Classification: Mathematical Physics

We apply general relativity theory to the problem of the gravitational collapse of a radiating superdense star. The physical problem requires the junction conditions at the boundary to be satisfied. This involves matching the curvature and matter components relating to the interior and exterior of the star at the boundary of the star. We obtain a highly nonlinear partial differential equation. In order to study physical features such as the temperature and the luminosity of the star we are required to solve the governing equation exactly. In this paper we show that particular transformations lead to exact models. These are the first models describing radiating collapse with shear, expansion and acceleration of the gravitating stellar fluid.
A Central Limit Theorem for integer partitions

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SAMS Subject Classification: Combinatorics and Graph Theory

Given a monotone increasing sequence $\lambda_k$ of positive integers, let $\omega_n$ be the length of a random partition of a positive integer $n$ into distinct members of the sequence $\lambda_k$. It is known that for sequences $\lambda_k$ satisfying certain technical conditions, the limit distribution of $\omega_n$ is Gaussian. We show that this is also true for the case of primes, i.e., $\lambda_k$ is set to be the $k$th prime number, for which those conditions are not fully satisfied. We will discuss how the result can be generalised to sequences of the form $\lambda_k = f(p_k)$ where $p_k$ is the $k$th prime number and $f(x)$ is a polynomial having integer values at integers. This includes powers of primes.
We discuss what spectral theory in Banach algebras is and how spectral theory is used in the theory of Banach algebras.
On the stochastic equation for second grade fluids

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SAMS Subject Classification: Probability Theory, Mathematical Statistics and Financial Mathematics

We investigate the stochastic equation for the motion of a second grade fluid filling a bounded domain of $\mathbb{R}^2$. Global existence of a weak probabilistic solution (and weak in the sense of partial differential equations) is expounded. We are also able to prove the pathwise uniqueness of these solutions. The two results yield the unique existence of a strong probabilistic solution. On this basis we show that under suitable conditions on the data we can construct a sequence of solutions of the stochastic second grade fluid that converges to the probabilistic weak solution of the stochastic Navier-Stokes equations when the physical parameter $\alpha$ tends to zero.
Tight maps and tight extension of metric spaces

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SAMS Subject Classification: Topology

A given extension \( e : X \to Y \) is tight provided that for any nonexpansive mapping \( f : Y \to Z \), if the composition \( fe \) is an extension then \( f \) is necessarily an extension. Given a metric space \( X \), there is always a tight extension \( e : X \to \varepsilon(X) \) such that \( \varepsilon(X) \) is maximal with that property. In other words, \( \varepsilon(X) \) does not have any proper tight extension. On the other hand, this extension is seen as a hyperconvex hull of \( X \), that is the minimal hyperconvex space (up to isomorphism) which contains \( X \). The construction of the extension \( e : X \to \varepsilon(X) \) has shown that \( \varepsilon(X) \) is the set of all tight maps or extremal functions defined on \( X \).

In this talk, the relations between the two concepts will be discussed.
Properties of predictor-corrector algorithms for variational inequalities

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SAMS Subject Classification: Computational Methods

Variational inequalities arise in many applications in the physical sciences and engineering, often in the context of weak or variational formulations of problems posed on convex sets rather than on subspaces of functions, and of problems involving differential inclusions rather than equations. The aim of this talk is to examine a class of predictor-corrector algorithms for the solution of the latter family of variational inequalities. These algorithms have been widely used, yet only partial results on their properties are available. Some new results on convergence will be presented, and illustrated in the context of a model problem.
Invocations on relational program calculi

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SAMS Subject Classification: Mathematical Aspects of Computer Science

The standard Galois connection [6, 3] between the relational [4] and predicate-
transformer [1] models of sequential programming (defined in terms of weakest
precondition) confers a certain similarity between them. This paper investiga-
tes the extent to which the important involution on transformers (which, for
instance, interchanges demonic and angelic nondeterminism, and reduces the
two kinds of simulation in the relational model to one kind in the transformer
model [5, 2]) carries over to relations. It is shown that no exact analogue exists;
that the two complement-based involutions are too weak to be of much use;
but that the translation to relations of transformer involution under the Galois
connection is just strong enough to support Boolean-algebra style reasoning, a
claim that is substantiated by proving properties of deterministic computations.
Throughout, the setting is that of the guarded-command language augmented
by the usual specification commands; and where possible algebraic reasoning is
used in place of the more conventional semantic reasoning.

References

    1976.
    Methods and their Comparison, Cambridge Tracts in Theoretical Computer
[3] G. Gierz, K.H. Hofman, K. Keimel, J.D. Lawson, M. Mislove and D. S. Scott,
[5] P.H.B. Gardiner and Carroll Morgan, A single complete rule for data refine-
    (55) 1944, p 494-513.
An integrated approach to community intervention, focusing on teacher and student development

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SAMS Subject Classification: History and Teaching of Mathematics

Maths education is under threat in the current conditions of South Africa. An integrated approach to community intervention will be required to sustain the educational system. It has been noted that the gap between tertiary and secondary education has widened, thus posing a threat to the calibre of students entering into the university. Only by the use of an Effective Community Intervention Programme (ECIP), will the gap between secondary and tertiary education have a possibility of being bridged. Students from urban and rural secondary educational backgrounds will have different circumstances in which education was presented to them, which adds to the difficulty of an Effective Outreach Programme (EOP). Although students have different backgrounds the same fundamental lack of knowledge is inherent in both the urban and rural students. Only through an integrated approach of both teacher and student development can we hope to address the gap between tertiary and secondary education. This conference proceedings, highlights the technique used by the University of Johannesburg for its Community Intervention Programme (CIP).
Some self orthogonal designs, self orthogonal codes and strongly regular graphs from the linear group $L_4(3)$

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SAMS Subject Classification: Algebra

We construct self-orthogonal codes obtained from the row span over $\mathbb{F}_2$ or $\mathbb{F}_3$ of the incidence (resp. adjacency) matrices of some self-orthogonal designs (resp. strongly regular graphs) defined by the action of the simple linear group $L_4(3)$ on the conjugacy classes of some of its maximal subgroups. We establish some properties of these codes and the nature of some classes of codewords, especially those of minimum and maximum weight. Further, we describe the structure of the stabilizers of minimum and maximum weight codewords and show that in many instances these are maximal subgroups of the automorphism groups of the corresponding codes or of $L_3(4)$. Some of the codes are optimal or near optimal for the given length and dimension. The dual codes of the graphs admit majority logic decoding.
Strong convergence of gradients for solutions of quasilinear stochastic parabolic equations

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SAMS Subject Classification: PDE’s and Dynamical Systems

For a given time horizon $[0, T]$, we consider the probability set-up $(\Omega, \mathcal{F}, \mathbb{P})$ on which is built a $d$–dimensional Brownian motion $W = (W^1_t, ..., W^d_t)$. Let $D$ be a bounded domain in the Euclidean space $\mathbb{R}^n (n \geq 2)$ and denote by $Q_T$ the cylinder $D \times (0, T)$. In $Q_T \times \Omega$, we consider the sequence of initial boundary value problems for the quasilinear stochastic parabolic equations

$$du_k + A(x, u_k, \nabla u_k) dt = f_k dt + g_k dW; \ k = 1, 2, ... \tag{1}$$

in the sense of distributions. Here $A$ is a Leray-Lions second-order monotone elliptic operator, $f_k$ (resp. $g_k$) are given functions (resp. $d$-dimensional vector-functions).

Under certain conditions, we prove that the sequences of gradient of the solutions $u_k$ and gradient of some truncations of $u_k$ converge strongly in some appropriate topologies as $k \to \infty$.

These results are motivated by issues arising in the emerging theory of homogenization of nonlinear stochastic evolution problems.
A group of the form $2^8:O^+_8(2)$

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SAMS Subject Classification: Algebra

The group $G = 2^8:O^+_8(2)$ is a maximal subgroup of $O^+_9(2)$ of index 527 and order 44590694400. The group $G$ has in turn two maximal subgroups $SP(6,2)$ and $2^6:A_8$ of index 120 and 135 respectively. The two maximal subgroups of $G$ together with $O^+_8(2)$ are also inertia factors of $2^8:O^+_8(2)$. Using these inertia factors and orthogonality rules we construct the Fischer - Clifford matrices of $2^8:O^+_8(2)$ which together with partial character tables we use to construct the character table of $2^8:O^+_8(2)$.
The closure of the smallest ideal of an ultrafilter semigroup

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SAMS Subject Classification: Topology

Let $S$ be a discrete semigroup, let $\beta S$ be the Stone-Čech compactification of $S$, and let $T$ be a closed subsemigroup of $\beta S$. We characterize ultrafilters from the smallest ideal $K(T)$ of $T$ and from its closure $\overline{\ell} K(T)$. We show that, for a large class of closed subsemigroups of $\beta S$, $\overline{\ell} K(T)$ is not an ideal of $T$. This class includes the subsemigroups $0^+ \subset \beta \mathbb{R}_d$ and $\mathbb{H}_\kappa \subset \beta(\bigoplus_\kappa \mathbb{Z}_2)$.

References

A new spectral-homotopy analysis method for solving nonlinear second order boundary value problems

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SAMS Subject Classification: Computational Methods

We present a spectral modification of the homotopy analysis method (HAM) for solving nonlinear second-order boundary value problems (BVPs). The implementation of the new approach is demonstrated by solving the Darcy-Brinkman-Forchheimer equation for steady fully developed fluid flow in a horizontal channel filled with a porous medium. The model equation is solved concurrently using the standard HAM approach and numerically using a shooting method based on the fourth order Runge-Kutta scheme. The results demonstrate that the new spectral homotopy analysis method is more efficient and converges faster than the standard homotopy analysis method.
Independent sets meeting all longest paths in oriented graphs

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SAMS Subject Classification: Combinatorics and Graph Theory

Laborde, Payan and Xuong [1] conjectured that every digraph has an independent set of vertices that meets (intersects) every longest path. We consider the conjecture for oriented graphs with small detour deficiency.

References

Domination partitions in graphs

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SAMS Subject Classification: Combinatorics and Graph Theory

A dominating set of a graph is a set of vertices such that every vertex not in the set is adjacent to a vertex in the set, a total dominating set of a graph is a set of vertices such that every vertex is adjacent to a vertex in the set, and a paired-dominating set of a graph is a dominating set such that the subgraph induced by the dominating set contains a perfect matching. A simple yet fundamental observation made by Ore [1] is that every graph of minimum degree at least one contains two disjoint dominating sets. In contrast to that, Zelinka [2, 3] showed that no minimum degree is sufficient to guarantee the existence of two disjoint total dominating sets (and hence also, two disjoint paired-dominating sets). In this talk, a number of results are presented which are somehow located between these observations. In particular, we show that the vertex set of every graph with minimum degree at least two and with no component that is a 5-cycle can be partitioned into a dominating set and a total dominating set. Furthermore, we present a constructive characterization of graphs whose vertex set can be thus partitioned. We then contrast similar results for partitioning the vertex set of a graph into a dominating set and a paired-dominating set.

References

Modelling the impact of immunosenescence on the dynamics of HIV-1 infection

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SAMS Subject Classification: ODE’s and Dynamical Systems

The severity of the numerous factors that influence the rate of HIV-1 disease progression leads to alterations in the observed clinical picture of HIV-1 infected individuals. Age of the human immune system at infection is one of such factors, whose impact on HIV-1’s pathogenesis has not been substantially evaluated. The time of emergence of HIV-1 associated opportunistic diseases also varies according to the patients immune regenerative capacity, which is partly dependent on age. Our principle objective is to determine the patient’s lifespan and account for the puzzling qualitative features of age-related immune deficiency associated with HIV-1 infection. A compartmental ordinary differential equation model describing the immunological changes in the concentration of CD4$^+$ and CD8$^+$ T lymphocytes, macrophages, stem-cell reservoir and virions in the blood is used to assess the dynamics of HIV-1 infection with respect to immunosenescence and to address the rapid exhaustion of the T-cell reservoir and the accelerated aging of the immune system. Variation of age at HIV-1 infection showed that older age ($\geq 50$ years) is linked to both a shortened interval between the onset of AIDS and death, and a very low virologic suppression compared to younger individuals ($\leq 30$ years). Incorporating anti-retroviral therapy indicated that its administration fails to increase the concentration of the haematopoietic stem-cell reservoir, but it strengthens the immune responsiveness towards opportunistic diseases (Tuberculosis). We conclude that: age at HIV-1 infection is a significant predictor of a patient’s lifespan; the quality of immune resources, the degree and speed of immune restoration is reduced in older patients; and lastly, elderly individuals rapidly progress to AIDS and die faster compared to younger individuals.
Structure of the multiplicative group of the field of $p$-adic numbers

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SAMS Subject Classification: Algebra

It is always in the interest of Mathematicians to develop different objects which do not necessarily have the same elements but with the same structure. By taking $\mathbb{Q}_p$, $\mathbb{Z}_p$, $\mathbb{Z}$, and $p$ to be the field of $p$-adic numbers, the ring of $p$-adic integers, the set of integers and a prime number respectively, we show that the group of units of $\mathbb{Q}_p^*$ is isomorphic to $\mathbb{Z} \times \mathbb{Z}_p \times \mathbb{Z}/(p-1)$ if $p \neq 2$ and to $\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}/2\mathbb{Z}$ if $p = 2$.

References
Semiprincipal closed ideals of $\beta S$

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SAMS Subject Classification: Topology

Let $S$ be an infinite discrete semigroup and let $\beta S$ be the Stone-Čech compactification of $S$. For every $p \in \beta S$, $\text{cl}((\beta S)p(\beta S))$ is a closed two sided ideal of $\beta S$ called the semiprincipal closed ideal generated by $p$. We show that if $S$ can be embedded into a group, then $\beta S$ contains $2^{2^{|S|}}$ pairwise incomparable semiprincipal closed ideals.
In this talk, we show that when the Frölicher smooth structure is induced on a product or a coproduct, there are three natural topologies underlying the resulting object. We study these topologies and compare them in each case. We show that on a product space the product topology is equal to the Frölicher topology. However, the three topologies are equal on the coproduct space. We end this paper by comparing topologies on the tangent bundle $TM$ of a Frölicher space $M$. 
Pancyclicity of $k$-traceable oriented graphs

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An oriented graph is $k$-traceable if each of its induced subdigraphs of order $k$ is traceable. Obviously, an oriented graph is 2-traceable if and only if it is a tournament. Thus, for $k \geq 3$, $k$-traceable oriented graphs are natural generalizations of tournaments. It is known that every nontrivial strong tournament is pancyclic.
Finite model properties for modal logics

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SAMS Subject Classification: Mathematical Logic and Foundations; Algebra

Modal logics are extensions of classical logic by an additional operator ♦ and some additional axioms. There are two natural classes of models for modal logic - algebraic and relational. The algebraic models are boolean algebras with additional operations of type \((A, \land, \lor, \neg, 0, 1, \diamondsuit)\); the relational models are of type \((W, R, V)\), where \(W\) is a set, \(R\) a binary relation on \(W\) and \(V\) a valuation that maps variables to subsets of \(W\). In proving the finite model property for a modal logic, one may construct finite models using either algebraic or relational methods (i.e., filtrations). In this talk we discuss the connections between these two methods of finite model construction and how to translate from one to the other.
Pseudo-topologies on reduced power algebras

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SAMS Subject Classification: Algebra; Topology

We show how pseudo-topologies, in particular convergence structures, may be defined in a natural way on reduced power algebras (RPA-s). As a first and most basic example, we give a construction of the set $\mathbb{R}$ of real numbers, and of its usual metric topology, as a quotient space of a suitable subset of $\mathbb{Q}^\mathbb{N}$. In particular, we show how the topology on $\mathbb{R}$ can be obtained as a quotient topology, with respect to a suitable topology on the set of Cauchy sequences from $\mathbb{Q}$. This construction serves as the prototype for pseudo-topologies on more general, and in particular non Archimedean, reduced power algebras. A major interest in such an extension of the usual concept of topology is in its convenient categorial properties which are unfortunately missing in the case of the usual topology. And such categorial properties turn out to lead in simple natural ways to important topological ones.
Integer programming-based constructions of Latin squares

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Integer programming models may be used to construct special types of Latin squares by appropriately defining the variables and constraints such that a feasible (or optimal) solution defines the desired Latin square. Recently Appa et al. [1, 2] used integer programming in order to solve certain problems concerning mutually orthogonal Latin squares, including a proof of the non-existence of a pair of mutually orthogonal Latin squares of order 6 by an attempted LP-based construction of such a pair. In this talk we consider the construction of various types of Latin squares by utilising integer programming as well as algebraic identities which define special types of Latin squares. These algebraic identities may be incorporated as constraints in an integer programming model in order to construct a quasi-group for which the multiplication table represents the desired Latin square. One such an example is a self-orthogonal Latin square (SOLS) (a Latin square orthogonal to its transpose), where the multiplication table of a quasi-group satisfying the identity $yx = x(xy)$ represents a SOLS.

References

Automorphism groups of generalized triangular matrix rings

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SAMS Subject Classification: Algebra

We call a ring strongly indecomposable if it cannot be represented as a non-trivial (i.e. \( M \neq 0 \)) generalized triangular matrix ring \( \left( \begin{array}{cc} R & M \\ 0 & S \end{array} \right) \), for some rings \( R \) and \( S \) and some \( R\text{-}S\)-bimodule \( RM_S \). Examples of such rings include rings with only the trivial idempotents 0 and 1, as well as endomorphism rings of vector spaces, or more generally, semiprime indecomposable rings. We show that if \( R \) and \( S \) are strongly indecomposable rings, then the triangulation of the non-trivial generalized triangular matrix ring \( \left( \begin{array}{cc} R & M \\ 0 & S \end{array} \right) \) is unique up to isomorphism; to be more precise, if \( \varphi : \left( \begin{array}{cc} R & M \\ 0 & S \end{array} \right) \to \left( \begin{array}{cc} R' & M' \\ 0 & S' \end{array} \right) \) is an isomorphism, then there are isomorphisms \( \rho : R \to R' \) and \( \psi : S \to S' \) such that \( \chi := \varphi|_M : M \to M' \) is an \( R\text{-}S\)-bimodule isomorphism relative to \( \rho \) and \( \psi \). In particular, this result describes the automorphism groups of such upper triangular matrix rings \( \left( \begin{array}{cc} R & M \\ 0 & S \end{array} \right) \).

References

A universal model of interaction

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SAMS Subject Classification: Mathematical Physics

A universal model of interaction is proposed with time as an independent coordinate. The dynamics of the model is determined by a Lagrangian. It leads to conservation equations of energy, total angular momentum and the \( z \) component of the angular momentum. These yield a Keplerian orbit in three dimensions, which gives the observed values of perihelion precession and bending of light by a massive object. An expression for gravitational redshift is derived by accepting the local validity of special relativity. Exact expressions for gravito-magnetic relations, as well as their associated Lorentz-type force, are derived. The model conforms to recent tests of higher order gravitational effects such as those measured by earth satellites and those of binary pulsars.

The model can be applied to derive the fine spectrum of hydrogen and the properties of the nuclear force.
We show that every minor of an $n \times n$ Laplace matrix, i.e., a symmetric matrix whose row- and column sums are 0, can be written in terms of those $\binom{n}{2}$ minors that are obtained by deleting two rows and the corresponding columns.

This identity has interesting applications to the enumeration of spanning trees. Specifically, we prove that if a subgraph of a graph $G$ is replaced by an electrically equivalent graph, the number of spanning trees only changes by a factor that does not depend on $G$. This allows us to employ techniques from the theory of electrical networks—such as the Wye-Delta transform—to determine the number of spanning trees of a graph, a technique that is particularly useful if the graphs under consideration are highly symmetric.

Furthermore, the identity also leads to a solution of the following inverse problem: Given an electrical network, the so-called effective resistance between any two vertices can be computed in terms of the given resistances. Our identity yields an explicit formula for resistances in terms of effective resistances.
A set $S$ of vertices in a graph $G$ is a total dominating set of $G$ if every vertex of $G$ is adjacent to some vertex in $S$. The minimum cardinality of a total dominating set is called the total domination number.

A hypergraph, $H$, contains a vertex set denoted by $V(H)$ and an edge set denoted by $E(H)$. Every hyper-edge in $H$ is a subset of the vertices. For graphs these sets all have size two, but for hypergraphs they can have any size. A transversal (also called a hitting set) in a hypergraph, $H$, is a set of vertices $T \subseteq V(H)$, such that every hyper-edge in $E(H)$ contains at least one vertex from $T$.

We will both give bounds on the size of transversals in several kind of hypergraphs and show how these bounds can be used to obtain many different kind of bounds for the total domination number of a graph with properties such as (i) minimum degree 3 or 4, (ii) 2-connected, (iii) minimum degree 2, containing no induced 6-cycles and (iv) minimum degree 3, containing no 4-cycle.

The area of fixed parameter tractable algorithms is new, interesting and growing rapidly. As finding transversals in 3-uniform hypergraphs (i.e. all edges contain 3 vertices) has many application, we will mention a fixed parameter tractable algorithm for this problem. This algorithm can immediately be used in areas such as computational biology (related to phylogenetic trees) and tournaments (finding a minimum feedback vertex set). The time complexity of our algorithm beats all previously know algorithms.

We finally mention several open problems and conjectures.
Idempotents in Stone-Čech compactifications and homogeneous maximal spaces

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SAMS Subject Classification: Topology

Let $G$ be an infinite group and let $\beta G$ be the Stone-Čech compactification of $G$ as a discrete semigroup. We take the points of $\beta G$ to be the ultrafilters on $G$. Being a compact right topological semigroup, $\beta G$ has idempotents. Every idempotent $p \in \beta G$ determines a left translation invariant Hausdorff topology $T_p$ on $G$ with a neighborhood base at the identity $e \in G$ consisting of subsets $A \cup \{e\}$ where $A \in p$. An idempotent $p \in \beta G$ is regular if $p$ is uniform ($= \text{ for every } A \in p, |A| = |G|$) and the topology $T_p$ is regular. We show that for every infinite group $G$, there exists a regular idempotent in $\beta G$. As a consequence we obtain that for every infinite cardinal $\kappa$, there exists a homogeneous regular maximal space of dispersion character $\kappa$, which is the answer to an old difficult question. Another consequence tells us that there exists a translation invariant regular maximal topology on the real line of dispersion character $\kappa$ stronger than the natural topology.
The conjugacy classes of a subgroup $S^m_n : C_m$ of $S_{mn}$

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SAMS Subject Classification: Algebra

Let $n, m$ be positive integers. Let $S^m_n$ be the direct product of $m$ copies of the symmetric group $S_n$ of degree $n$. Then $S^m_n$ is a subgroup of the symmetric group $S_{mn}$ of degree $m \times n$. Let $g \in S_{mn}$, of type $[m^n]$ where each $m$-cycle contains one symbol from each set of symbols in that order on which the copies of $S_n$ act. Let $C_m = \langle g \rangle$ of order $m$. The wreath product of $S_n$ with $C_m$ is a split extension of $S^m_n$ by $C_m$, denoted by $S^m_n : C_m$. Indeed $S^m_n : C_m$ is a subgroup of the symmetric group $S_{mn}$. In [1], a method for constructing the conjugacy classes of the groups $S^m_n : C_m$ where $m$ is prime, is given. In this paper that method is extended to any positive integer $m$.

References
Self-adjoint fourth order differential operators with eigenvalue parameter dependent boundary conditions

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SAMS Subject Classification: Operator Algebra and Functional Analysis

We consider the eigenvalue problem

\[ y^{(4)}(\lambda, x) - (gy')'(\lambda, x) = \lambda^2 y(\lambda, x) \]

with separated boundary conditions

\[ B_j(\lambda) y = 0 \quad \text{for} \quad j = 1, \ldots, 4, \]

where \( g \in C^1[0, a] \) is a real valued function, \( B_j(\lambda) y = y^{[p]}(a_j) \) or \( B_j(\lambda) y = y^{[p]}(a_j) + i\varepsilon_j \alpha \lambda y^{[q]}(a_j) \), \( a_j = 0 \) for \( j = 1, 2 \) and \( a_j = a > 0 \) for \( j = 3, 4 \), \( \alpha > 0 \), \( \varepsilon_j \in \{-1, 1\} \). We will associate to the above eigenvalue problem a quadratic operator pencil

\[ L(\lambda) = \lambda^2 M - i\alpha \lambda K - A \]

in the space \( L^2(0, a) \oplus \mathbb{C}^k \), where \( M = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \) and \( K = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \) are bounded self-adjoint operators and \( k \) is the number of boundary conditions which depend on \( \lambda \). We give necessary and sufficient conditions for the operator \( A \) to be self-adjoint.