

Involutions on relational program calculi

I.M. Rewitzky*

University of Stellenbosch

rewitzky@sun.ac.za

J.W. Sanders

International Institute for Software Technology, Macao

SAMS Subject Classification: Mathematical Aspects of Computer Science

The standard Galois connection [6, 3] between the relational [4] and predicate-transformer [1] models of sequential programming (defined in terms of weakest precondition) confers a certain similarity between them. This paper investigates the extent to which the important involution on transformers (which, for instance, interchanges demonic and angelic nondeterminism, and reduces the two kinds of simulation in the relational model to one kind in the transformer model [5, 2]) carries over to relations. It is shown that no exact analogue exists; that the two complement-based involutions are too weak to be of much use; but that the translation to relations of transformer involution under the Galois connection is just strong enough to support Boolean-algebra style reasoning, a claim that is substantiated by proving properties of deterministic computations. Throughout, the setting is that of the guarded-command language augmented by the usual specification commands; and where possible algebraic reasoning is used in place of the more conventional semantic reasoning.

References

- [1] E.W. Dijkstra, *A Discipline of Programming*, Prentice-Hall International 1976.
- [2] W.-P. de Roever and K. Engelhardt, *Data Refinement: Model-Oriented Proof Methods and their Comparison*, Cambridge Tracts in Theoretical Computer Science, Cambridge University Press 1998.
- [3] G. Gierz, K.H. Hofman, K. Keimel, J.D. Lawson, M. Mislove and D. S. Scott, *A Compendium of Continuous Lattices*, Springer Verlag 1980.
- [4] C.A.R. Hoare et al., *The laws of programming*, Communications of the ACM (30)(8) 1987, p 672686.
- [5] P.H.B. Gardiner and Carroll Morgan, *A single complete rule for data refinement*, Formal Aspects of Computing (5)(4) 1993, p 367382.
- [6] O. Ore, *Galois connexions*, Transactions of the American Mathematical Society (55) 1944, p 494-513.