

Results on spectral continuity in ordered Banach algebras

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Since J. D. Newburgh initiated the subject of spectral continuity in Banach algebras in 1951, it has been studied extensively. However, spectral continuity in the context of an ordered Banach algebra (OBA) has only been studied in recent years. Newburgh showed that the spectrum and spectral radius functions are upper semicontinuous. However, a well-known example by S. Kakutani illustrates that these functions are not continuous. In fact, this example shows that, in the case of an OBA, the spectrum and spectral radius functions are not even continuous on the algebra cone.

In this talk we will exhibit a number of theorems illustrating spectral continuity properties of positive elements. Among other things, we show (under natural conditions) that if a is a positive element, then the restriction of the spectral radius function to the set $C(a) = \{x : a \leq x \text{ and } (ax \leq xa \text{ or } xa \leq ax)\}$ is continuous at a (see [1]).

Given an element a of a Banach algebra A , the boundary spectrum of a , denoted by $S_{\partial}(a)$, was defined in [2] as the set of all $\lambda \in \mathbb{C}$ such that $\lambda - a$ is an element of the boundary of the set of all non-invertible elements of A . Using this concept, we show that if a is a positive element (relative to a closed and normal algebra cone C in an OBA) such that $S_{\partial}(a) \cap \mathbb{R}^+$ consists of the spectral radius of a only, then the restriction of the spectral radius function to C is continuous at a (see [3]).

We also consider some applications.

References

- [1] S. Mouton, *On spectral continuity of positive elements*, *Studia Math.* 174 (2006), 75–84.
- [2] S. Mouton, *On the boundary spectrum in Banach algebras*, *Bull. Austral. Math. Soc.* 74 (2006), 239–246.
- [3] S. Mouton, *A condition for spectral continuity of positive elements*, *Proceedings of the American Mathematical Society* 137(5) 2009, 1777–1782.