

# MTL-chains with modalities

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We discuss modal extensions of the logic MTL, or *monoidal t-norm logic*, which is a many-valued logic that may be considered the logic of ‘left-continuous t-norms’. A class of algebraic models for MTL consists of all *MTL-chains*, which are bounded chains in the lattice-sense, together with a t-norm operation  $\circ$  and associated residual operation  $\rightarrow$ .

By adding modal operators to MTL we wish to increase its expressive power. Natural examples of such modal operators are the linear logic exponential  $!$  and the Baaz delta. Algebraic models for modal MTL logics are then *modal MTL-chains* which are MTL-chains with an additional operation. We are interested in completions of modal MTL-chains and the equations that are preserved by such completions. In particular, we seek a Sahlqvist-like theory for modal MTL-chains analogous to the Sahlqvist theory for modal algebras, i.e., a syntactic descriptions of equations that are preserved by completion. As a special case, we consider the logic  $LK^r$ , which is essentially MTL with a linear logic modality, as defined in [2]. We also consider negative modalities such as  $\sim$ , which are order-reversing and are natural forms of negation.

## References

- [1] C. van Alten, *Preservation Theorems for MTL-chains*, Manuscript.
- [2] A. Ciabattoni, G. Metcalfe and F. Motagna, *Adding Modalities to MTL and its Extensions*, To appear in the Proceedings of the Linz Symposium 2005.