Centralizers of matrices

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Since the minimum polynomial and characteristic polynomial of a non-scalar 2×2 matrix in $M_2(F)$, the full 2×2 matrix ring over the field F, coincide, the centralizer in $M_2(F)$ of a 2×2 matrix is known from the more general result, Theorem 5 on page 23, in [2]. As a corollary of this result a concrete description of the centralizer of a 2×2 matrix over an integral domain D in $M_2(D)$ can be derived.

In this talk the concept of a k-matrix in $M_2(R/\langle k\rangle)$, where R is an arbitrary unique factorization domain (UFD) and k an arbitrary nonzero nonunit in R, is introduced. We obtain a concrete description of the centralizer of a k-matrix $\widehat{B}$ in $M_2(R/\langle k\rangle)$ as the sum of two subrings $\mathcal{S}_1$ and $\mathcal{S}_2$ of $M_2(R/\langle k\rangle)$, where $\mathcal{S}_1$ is the image (under the induced epimorphism from $M_2(R)$ to $M_2(R/\langle k\rangle)$) of the centralizer in $M_2(R)$ of a pre-image of $\widehat{B}$, and where the entries in $\mathcal{S}_2$ are intersections of certain annihilators of elements arising from the entries of $\widehat{B}$. It turns out that if R is a principal ideal domain (PID) then every matrix in $M_2(R/\langle k\rangle)$ is a k-matrix for every k. However, this is not the case for UFD's in general. Moreover, for every factor ring $R/\langle k\rangle$ with zero divisors and every $n \geq 3$ there is a matrix for which the above mentioned description is not valid.

References

[1] T.W. Hungerford, *Algebra,* Springer-Verlag, New York, 1974.
[2] D.A. Suprenenko and R.I. Tyshkevich, *Commutative Matrices,* Academic Press, New York, 1968.