Controlling a stopped diffusion process to reach a goal

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We consider a problem of optimally controlling a two-dimensional diffusion process

$$\begin{aligned} dx_t^{\mu,\beta} &= \mu(x_t)dt + \beta(x_t)dB_t^1; \quad x(0) = x, \\ dy_t &= \alpha y_t dt + (\sigma\sqrt{x_t} + \gamma)y_t dB_t^2; \quad y(0) = y, \end{aligned}$$

initially starting in the interior of a domain $D_\varphi = \{(x, y) \in \mathbb{R}_+^2 : \varphi(x) < y < \theta\varphi(x)\}$ until it reaches the line $y = \theta\varphi(x)$ at a stopping time $\tau \leq T_0$, where T_0, $\alpha, \sigma, \gamma > 0$ and $\theta > 1$ are fixed positive constants and $\varphi(x)$ is a given positive strictly increasing, twice continuously differentiable function on $(0, \infty)$. The goal is to maximize the probability criterion

$$\sup_{(\mu,\beta)\in\mathcal{M}(x)} \mathrm{P}\left(y_\tau = \theta\varphi(x_\tau^{\mu,\beta}), \quad \tau \leq T_0 | x(0) = x, y(0) = y\right), \quad x, y \in D_\varphi$$

over a class of admissible controls $\mathcal{M}(x)$ consisting of bounded, Borel measurable functions. Under suitable conditions, it is shown that the maximal probability is given explicitly and the optimal process is determined explicitly by

$$\rho(\varphi(x), \varphi'(x), \varphi''(x)) = \sup\left\{\frac{\mu(x)\varphi'(x) - (\alpha - \frac{1}{2}(\sigma\sqrt{x} + \gamma)^2)\varphi(x)}{\beta(x)^2} : (\mu, \beta) \in \mathcal{M}(x)\right\}.$$