Convergence of ray sequences of Padé approximants for a class of hypergeometric functions

K. Jordaan*
University of Pretoria
kjordaan@up.ac.za
K. Driver
University of Cape Town
Kathy.Driver@uct.ac.za

SAMS Subject Classification: Real and Complex Analysis

The denominator polynomial $Q_{mn}(z)$ in the $[m/n]$ Padé approximant $P_{mn}(z)/Q_{mn}(z)$ for ${}_2F_1(a,1;c;z)$ was explicitly evaluated by Padé for $m \geq n-1$ and $c \notin \mathbb{Z}^-$. We show that for $c > a > 0$ and $m \geq n-1$, the poles of $P_{mn}(z)/Q_{mn}(z)$ lie on the cut $(1,\infty)$. We deduce that the sequence of approximants $P_{mn}(z)/Q_{mn}(z)$ converges to ${}_2F_1(a,1;c;z)$ as $m \to \infty$, $n/m \to \rho$ with $0 < \rho \leq 1$, uniformly on compact subsets of the unit disc $|z| < 1$ for $c > a > 0$.