Lattice-valued categories of lattice-valued convergence spaces

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In this talk we have another look at the category $SL\text{-}GCS$ of lattice-valued generalized convergence spaces [2, 3]. We show that extending the structure of continuous convergence (which makes $SL\text{-}GCS$ a cartesian closed category) from the set of continuous mappings between spaces to a set F of arbitrary mappings between spaces, one of the axioms satisfied by the objects in $SL\text{-}GCS$ may no longer be valid for F. This poses the question: "How far is F away from being in $SL\text{-}GCS$?" Using a frame as lattice, this question can be answered if we attach "grades of continuity" to the mappings in F. In this way, we are naturally led to the concept of a lattice-valued category in the sense of Šostak [4, 5, 6]. Such an L-category consists of an ordinary category [1] of "potential objects" and "potential morphisms" together with two L-classes, assigning a grade of being an object and of being a morphisms of the L-category. We describe initial constructions and function spaces of the resulting L-category of L-convergence spaces. Also we use Šostak's concept of L-category and study "how far away a lattice-valued convergence space is from being a lattice-valued topological space".

References

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