

Fredholm theory in Banach algebras

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SAMS Subject Classification: Operator Algebra and Functional Analysis

Fredholm operators were originally defined in an algebra of bounded linear operators on a Banach space. In [2], R.E. Harte used Atkinson's theorem to generalise this definition to Fredholm elements in a Banach algebra relative to some fixed bounded Banach algebra homomorphism. Weyl and Browder elements were also introduced and results related to elements with these properties were obtained. Later the boundedness of the Banach algebra homomorphism was dropped from the definition and most results could still be obtained as H. du T. Mouton and H. Raubenheimer showed in [3], using a result of J.J. Grobler and H. Raubenheimer in [1]. In this talk the definitions of Fredholm, Weyl and Browder elements in a Banach algebra are given. Some inclusion properties of the sets of Fredholm, Weyl and Browder elements in relation to each other are shown, as well as some spectral inclusion properties. Most of these results in their original form are due to Harte. Finally, some properties of upper and lower semi-regularities due to V. Müller ([4]) are applied to some sets related to Fredholm Theory in Banach algebras.

References

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