Strong transversals in hypergraphs and double total domination in graphs

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Let $H$ be a 3-uniform hypergraph of order $n$ and size $m$ and let $T$ be a subset of vertices of $H$. The set $T$ is a strong transversal in $H$ if $T$ contains at least two vertices from every edge of $H$. The strong transversal number $\tau_s(H)$ of $H$ is the minimum size of a strong transversal in $H$. We show that $7\tau_s(H) \leq 4n+2m$ and we characterize the hypergraphs that achieve equality in this bound. In particular, we show that the Fano plane is the only connected 3-uniform hypergraph $H$ of order $n \geq 6$ and size $m$ that achieves equality in this bound.

A set $S$ of vertices in a graph $G$ is a double total dominating set of $G$ if every vertex of $G$ is adjacent to at least two vertices in $S$. The minimum cardinality of a double total dominating set of $G$ is the double total domination number $\gamma_{2,\tau}(G)$ of $G$. Let $G$ be a connected graph of order $n$ with minimum degree at least three. Using our hypergraph results, we show that $\gamma_{2,\tau}(G) \leq 6n/7$ with equality if and only if $G$ is the Heawood graph.