

Strong transversals in hypergraphs and double total domination in graphs

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Let H be a 3-uniform hypergraph of order n and size m and let T be a subset of vertices of H . The set T is a strong transversal in H if T contains at least two vertices from every edge of H . The strong transversal number $\tau_s(H)$ of H is the minimum size of a strong transversal in H . We show that $7\tau_s(H) \leq 4n+2m$ and we characterize the hypergraphs that achieve equality in this bound. In particular, we show that the Fano plane is the only connected 3-uniform hypergraph H of order $n \geq 6$ and size m that achieves equality in this bound. A set S of vertices in a graph G is a double total dominating set of G if every vertex of G is adjacent to at least two vertices in S . The minimum cardinality of a double total dominating set of G is the double total domination number $\gamma_{\times 2,t}(G)$ of G . Let G be a connected graph of order n with minimum degree at least three. Using our hypergraph results, we show that $\gamma_{\times 2,t}(G) \leq 6n/7$ with equality if and only if G is the Heawood graph.