# Binary codes and partial permutation decoding sets from the Johnson graphs

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The Johnson graph, denoted by $J(n, k)$, is the graph of which the vertex-set is the set of all $k$-subsets of $\Omega = \{1, 2, \ldots, n\}$, and any two vertices $u$ and $v$ constitute an edge $[u, v]$ if and only if $|u \cap v| = k - 1$. In this talk the codes and their duals generated by the adjacency matrix of $J(n, k)$ will be described. It will be shown that in each case, the code has a basis comprising minimum weight vectors. The same does not apply to the dual codes, since if both $n$ and $k$ are even, then the minimum weight vectors do not span the dual code.

The codes from $J(n, k)$ are also the codes of the $1-(\binom{n}{k}, k(n-k), n(n-k))$ design $\mathcal{D}$ which has the vertices of $J(n, k)$ and the supports of the incidence vectors of its adjacency matrix as its point-set $\mathcal{P}$ and its block-set $\mathcal{B}$ respectively.. It is known that the automorphism group of $J(n, k)$ is $S_n$, and it is shown by an explicit argument that the automorphism group of the code is also $S_n$ for $k > 2$, except when $k$ is odd and $n$ is even, in which case it is $S_{\binom{n}{k}}$. 3-PD-sets are obtained for the code in the case that $k \geq 4$ and $n \geq 8$ are even and $n \geq 2k$, and in the case that $k \geq 6$ is even and $n > 2k$ is odd.