A graph-theoretic proof of the non-existence of self-orthogonal Latin squares of order 6

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The non-existence of a pair of mutually orthogonal Latin squares of order six is a well-known result in the theory of combinatorial designs. It was conjectured by Euler in 1782 and was first proved by Tarry [4] in 1900 by means of an exhaustive enumeration of equivalence classes of Latin squares of order six. Various further proofs have since been given [1, 2, 3, 5], but these proofs generally require extensive prior subject knowledge in order to follow them, or are 'blind' proofs in the sense that most of the work is done by computer or by exhaustive enumeration. In this talk we present a graph-theoretic proof of a somewhat weaker result, namely the non-existence of self-orthogonal Latin squares of order six, by introducing the concept of a self-orthogonal Latin square graph. The advantage of this proof is that it is easily verifiable and accessible to discrete mathematicians not intimately familiar with the theory of combinatorial designs. The proof also does not require any significant prior knowledge of graph theory.

References

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