

Spectral characterizations of scalars in a Banach algebra

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For a complex Banach algebra A with unit $\mathbf{1}$, we give several characterizations of the scalars, that is, multiples of the identity. To a large extent, this work is a continuation and generalization of the work done on characterizations of the radical in Banach algebras. In particular it is shown that if $a \in A$ has the property that the number of elements in the spectrum of ax is less than or equal to the number of elements in the spectrum of x for all x in an arbitrary neighbourhood of $\mathbf{1}$, then a is a scalar. Moreover, as a consequence of some of the results, new spectral characterizations of commutative Banach algebras are obtained. In particular, A is commutative if and only if it has the property that the number of elements in the spectrum remains invariant under all permutations of three elements in some neighbourhood of the identity.