Hopf bifurcation of tuberculosis transmission model with vaccination

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We consider a deterministic SEI model for a transmission of tuberculosis. The model incorporates vaccination for newborns in which only a proportion of them are vaccinated at birth. We analyse the model mathematically and numerically. Bifurcation of multiple endemic equilibrium points exist as we varied a reinfection term. For this, a threshold value of the reinfection rate is set resulting in different kinds of bifurcation. Thus the system exhibits a backward and forward bifurcation having hopf bifurcation (as result of inherited periodic solutions) depending on the reinfection rate chosen. For certain chosen set of parameters, two endemic equilibriums exist below $R_0 = 1$, in which neither of them are stable. Thus it can be suggested that, disease free equilibrium (DFE) is globally stable for $R_0 < 1$. This result is different compared to the usual backward bifurcations, as the co-stability of the disease free and endemic equilibrium points has been discussed widely in other studies. While for R_0 above unity, the system has three endemic equilibrium points up to certain value of R_0 and onwards only one. From the three, only one endemic equilibrium point is stable and the DFE is unstable. The numerical results also reveal that, vaccination is of less importance if the waning effect is comparably high. It implies that treatment has to target unvaccinated individuals or boost vaccination has to be used. This is mainly because of high transition of vaccinated individuals to the respective unvaccinated compartments as protection of the vaccine wanes.