

## CLASSICAL AND QUANTUM COMPUTING

### EXERCISE VIII

We use (Ons maak gebruik van)  $\{|0\rangle, |1\rangle\}$  as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Find all (Vind alle)  $x, y, z \in \{0, 1, 2, 3\}$  such that (sodat)  $x + y + z = 0 \pmod{2}$ . What are the possible values of (Wat is die moontlike waardes van)

$$f(x, y, z) := \frac{(x + y + z) \pmod{4}}{2}.$$

when the condition holds (as die eerste vergelyking bevredig is). Now use the binary representation for (Gebruik nou die binêre voorstelling van)  $x = x_1x_0$ ,  $y = y_1y_0$  and (en)  $z = z_1z_0$  where (waar)  $x_0, x_1, y_0, y_1, z_0, z_1 \in \{0, 1\}$ . Describe the condition (Beskryf die vergelyking)  $x + y + z = 0 \pmod{2}$  in terms of (in terme van)  $x_0, x_1, y_0, y_1, z_0$  and (en)  $z_1$ . We find that (Ons vind dat)

$$f(x, y, z) = x_1 \oplus y_1 \oplus z_1 \oplus (x_0 + y_0 + z_0).$$

XOR is denoted by “ $\oplus$ ” and OR is denoted by “+”. (XOR word as “ $\oplus$ ” geskryf en OR word as “+” geskryf.) We use the mapping (Ons definieer die funksie)

$$f_1(0) = I \quad f_1(1) = U_H$$

Thus we can map from the triple  $(x_0, y_0, z_0)$  to linear operators acting on three qubits (Definieer die funksie van die drietal  $(x_0, y_0, z_0)$  na lineêre transformasies):

$$f_3(x_0, y_0, z_0) = f_1(x_0) \otimes f_1(y_0) \otimes f_1(z_0).$$

Let (Laat)

$$|\psi\rangle := \frac{1}{2}(|000\rangle - |011\rangle - |101\rangle - |110\rangle).$$

For each triple (Vir elk van die)  $(x_0, y_0, z_0)$  found in the first part of the question, calculate (wat in die eerste deel gevind is, bereken)

$$|\phi\rangle := f_3(x_0, y_0, z_0)|\psi\rangle.$$

Let (laat)  $s_x, s_y, s_z$  denote the result (0 or 1) of measuring the first, second and third qubit of  $|\phi\rangle$  respectively in the computational basis (die resultaat wees wanneer die eerste, tweede en derde qubit van  $|\phi\rangle$  in die basis van berekening gemeet word). In each case determine (Vir elke geval bepaal)

$$s_x + s_y + s_z \text{ mod } 2, \quad x_0 + y_0 + z_0.$$

2) Consider the following (Beskou die volgende)

$$|\psi\rangle := a|0\rangle + b|1\rangle, \quad |a|^2 + |b|^2 = 1$$

$$|\phi\rangle := |\psi\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Show that (Toon aan dat)  $|\phi\rangle$  can be written as (kan soos volg geskryf word)

$$\begin{aligned} |\phi\rangle &= \frac{1}{2\sqrt{2}}(|00\rangle + |11\rangle) \otimes (a|0\rangle + b|1\rangle) + \frac{1}{2\sqrt{2}}(|00\rangle - |11\rangle) \otimes (a|0\rangle - b|1\rangle) \\ &+ \frac{1}{2\sqrt{2}}(|01\rangle + |10\rangle) \otimes (a|1\rangle + b|0\rangle) + \frac{1}{2\sqrt{2}}(|01\rangle - |10\rangle) \otimes (a|1\rangle - b|0\rangle) \end{aligned}$$

Describe how measurement of the first two qubits can be used to obtain  $|\psi\rangle$  as the last qubit. (Beskryf hoe meting van die eerste twee qubits gebruik kan word om  $|\psi\rangle$  vir die laaste qubit te vind.)