CLASSICAL AND QUANTUM COMPUTING
EXERCISE V

1) Calculate the eigenvalues and eigenvectors of (Bereken die eiewaardes en eievektore van)

$$\frac{d^2}{dx^2}.$$

Consider the problem of a particle in a one-dimensional box. Beskou die probleem van 'n partikel in 'n een-dimensionele houer. The underlying Hilbert space is $L_2(-a,a)$. Die onderliggende Hilbertruimte is $L_2(-a,a)$. Solve the Schrödinger equation (Los die Schrödingervergelyking)

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

where (waar) $\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dq^2}$ as follows: (as volg op:) The formal solution is given by

$$\psi(t) = \exp(-i\hat{H}t/\hbar)\psi(0).$$

Expand $\psi(0)$ with respect to the eigenfunctions of the operator $\hat{H}$. The eigenfunctions

$$B = \{u_n^{(+)}, u_n^{(-)} \quad n \in \mathbf{N}\}$$

$$u_n^{(+)} = \frac{1}{\sqrt{a}}\cos\left(\frac{(n-\frac{1}{2})\pi q}{a}\right), \qquad u_n^{(-)} = \frac{1}{\sqrt{a}}\sin\left(\frac{n\pi q}{a}\right)$$

form an orthonormal basis of the Hilbert space. Brei $\psi(0)$ met betrekking tot die eiefunksies van die operator $\hat{H}$ uit. Die eiefunksies vorm 'n basis in die Hilbertruimte. Then apply $\exp(-i\hat{H}t/\hbar)$. Pas dan $\exp(-i\hat{H}t/\hbar)$ toe. Calculate the propability (Bereken die waarskynlikheid)

$$P = |\langle\phi, \psi(t)\rangle|^2$$

where (waar)

$$\phi(q) = \frac{1}{\sqrt{a}} \sin\left(\frac{\pi q}{a}\right)$$

and (en)

$$\psi(q,0) = \frac{1}{\sqrt{a}} \sin\left(\frac{\pi q}{a}\right)$$

2) Consider (Beskou)

$$A := i|0\rangle\langle 1| - i|1\rangle\langle 0|,$$

where (waar)

$$\{|0\rangle, |1\rangle\}$$

is an orthonormal basis in a 2-dimensional Hilbert space (is 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte).
Is A self adjoint ? (Is A self toegevoegde ?) Determine the eigenvalues and corresponding normalized eigenvectors of A (Bereken die eiewaardes en oorstemmende genormaliseerde eievektore). Calculate (Bereken)

$$U(t) = \exp(-iAt/\hbar).$$

For what values of t does $U(t)$ perform the NOT operation (Gee die waardes van t sodat $U(t)$ die NOT bewerking implementeer)

$$U(t)|0\rangle \to |1\rangle$$

$$U(t)|1\rangle \to |0\rangle$$

Calculate (Bereken) $U(t = \pi\hbar/4)$ and (en) $(U(t = \pi\hbar/4))^2$.