CLASSICAL AND QUANTUM COMPUTING
EXERCISE XX

1. Consider the state

$$|x\rangle := \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

in the Hilbert space $A \otimes B$, where $A = B = \mathbf{C}^2$. Calculate

$$\rho_A := \mathrm{tr}_B(|x\rangle\langle x|)$$

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$$-\mathrm{tr}(\rho_A \log_2 \rho_A)$$

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2. Consider the state

$$|x\rangle := \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

in the Hilbert space $A \otimes B$, where $A = B = \mathbf{C}^2$. Calculate

$$\rho_A := \mathrm{tr}_B(|x\rangle\langle x|)$$

$$-\mathrm{tr}(\rho_A \log_2 \rho_A).$$

3. Consider the state

$$|x\rangle := \frac{1}{2}U_1 \otimes U_2 \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

in the Hilbert space $A \otimes B$, where $A = B = \mathbf{C}^2$ and U_1 and U_2 are unitary matrices acting on $\mathbf{C}^2$. Calculate

$$\rho_A := \mathrm{tr}_B(|x\rangle\langle x|)$$

$$-\mathrm{tr}(\rho_A \log_2 \rho_A).$$

4. Consider the state

$$|x\rangle := \frac{1}{\sqrt{2}} U_1 \otimes U_2 \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

in the Hilbert space $A \otimes B$, where $A = B = \mathbf{C}^2$ and U_1 and U_2 are unitary matrices acting on $\mathbf{C}^2$. Calculate

$$\rho_A := \mathrm{tr}_B(|x\rangle\langle x|)$$

$$-\mathrm{tr}(\rho_A \log_2 \rho_A).$$