CLASSICAL AND QUANTUM COMPUTING

EXERCISE XI

We use (Ons maak gebruik van) $\{|0\rangle, |1\rangle\}$ as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Show that (Toon aan dat) U_{QFT} is unitary ('n unitêre transformasie is) . In other words show that (Met ander woorde toon aan dat) $U_{QFT}U_{QFT}^* = I$ where (waar)

$$U_{QFT} := \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} \sum_{k=0}^{2^n-1} e^{-i2\pi kj/2^n} |k\rangle\langle j|,$$

$$I := \sum_{j=0}^{2^n-1} |j\rangle\langle j|.$$

2) Show that (Toon aan dat) U_{IA} is unitary ('n unitêre transformasie is). In other words show that (Met ander woorde toon aan dat) $U_{IA}U_{IA}^* = I$ where (waar)

$$U_{IA} := \sum_{j=0}^{2^n-1} \sum_{k=0}^{2^n-1} \left(\frac{2}{2^n} - \delta_{jk} \right) |k\rangle\langle j|,$$

$$I := \sum_{j=0}^{2^n-1} |j\rangle\langle j|.$$

3) Suppose that the only errors which can occur to three qubits are described by the transforms (Neem aan dat die enigste foute wat voor kan kom in die drie qubits word deur die volgende transformasies beskryf)

$$\{I \otimes I \otimes I, I \otimes U_{NOT} \otimes U_{NOT}, I \otimes U_P \otimes U_P, I \otimes (U_P U_{NOT}) \otimes (U_P U_{NOT})\}.$$

where (waar)

$$U_P = |0\rangle\langle 0| - |1\rangle\langle 1|.$$

Describe how an arbitrary error (Beskryf hoe 'n algemene fout)

$$\alpha I \otimes I \otimes I + \beta I \otimes U_{NOT} \otimes U_{NOT} + \delta I \otimes U_P \otimes U_P + \gamma I \otimes (U_P U_{NOT}) \otimes (U_P U_{NOT})$$

on the state (op die staat)

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \otimes |\psi\rangle$$

can be corrected to obtain the correct $|\psi\rangle$ as the last qubit (reggestel kan word om $|\psi\rangle$ as die laaste qubit te vind), where (waar)

$$|\psi\rangle := a|0\rangle + b|1\rangle, \quad |a|^2 + |b|^2 = 1.$$