CLASSICAL AND QUANTUM COMPUTING
EXERCISE X

We use (Ons maak gebruik van) $\{|0\rangle, |1\rangle\}$ as an orthonormal basis for a 2-dimensional Hilbert space in the following questions (vir 'n ortonormale basis vir 'n 2-dimensionele Hilbertruimte in die volgende vrae).

1) Calculate the following in terms of (Bereken die volgende in terme van) I, X, Y, Z

$$\begin{array}{ll} a) & XZ \\ b) & ZX \\ c) & U_{CNOT}(X \otimes I)U_{CNOT} \\ d) & U_{CNOT}(I \otimes X)U_{CNOT} \\ e) & U_{CNOT}(Z \otimes I)U_{CNOT} \\ f) & U_{CNOT}(I \otimes Z)U_{CNOT} \\ g) & U_{CNOT}(X \otimes X)U_{CNOT} \\ h) & U_{CNOT}(Z \otimes Z)U_{CNOT} \\ i) & U_{CNOT}U_{CNOT} \end{array}$$

where (waar)

$$\begin{aligned} I &= |0\rangle\langle 0| + |1\rangle\langle 1| \\ X &= |0\rangle\langle 1| + |1\rangle\langle 0| \\ Y &= |0\rangle\langle 1| - |1\rangle\langle 0| \\ Z &= |0\rangle\langle 0| - |1\rangle\langle 1| \end{aligned}$$

2) Alice and Bob share n entangled pairs of the form (Alice en Bob deel n verstrikte pare) $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Thus we can write their shared state of $2n$ qubits in the form

(Dus is dit moontlik om die gedeelde staat van die $2n$ qubits soos volg te skryf)

$$|\psi\rangle = \frac{1}{\sqrt{2^n}} \sum_{j=0}^{2^n-1} |j\rangle \otimes |j\rangle$$

where the first n qubits belong to Alice and the second n qubits belong to Bob (waar die eerste n qubits aan Alice behoort, en die tweede n qubits aan Bob behoort). Furthermore Alice has 2^n bits (Alice het 2^n bisse) $a_0, \ldots, a_{2^n-1}$ and Bob has 2^n bits (en Bob het 2^n bisse) $b_0, \ldots, b_{2^n-1}$. Calculate (Bereken)

$$|\phi\rangle := U_{PA} \otimes U_{PB} |\psi\rangle$$

$$\left(\bigotimes_n U_H \right) \otimes \left(\bigotimes_n U_H \right) |\phi\rangle.$$

where U_{PA} and U_{PB} acts on the computational basis as follows (waar U_{PA} en U_{PB} soos volg op die basis van berekening toegepas word)

$$U_{PA}|j\rangle = (-1)^{a_j}|j\rangle, \qquad j = 0, 1, \ldots, 2^n - 1$$

$$U_{PB}|j\rangle = (-1)^{b_j}|j\rangle, \qquad j = 0, 1, \ldots, 2^n - 1$$

For each of the cases (Vir die gevalle)

$$a) \quad a_0 = b_0,\ a_1 = b_1\ , \ldots,\ a_{n-1} = b_{n-1}$$

$$b) \quad \sum_{k=0}^{2^n-1} |a_k - b_k| = 2^{n-1}$$

determine when measurement of the first n qubits in the computational basis yields the same result as measurement of the second n qubits in the computational basis (bepaal wanneer meting van die eerse n qubits in die basis van berekening presies dieselfde resultaat gee as meting van die tweede n qubits in die basis van berekening).