CLASSICAL AND QUANTUM COMPUTING

EXERCISE I

1) Consider the Hilbert space $L_2[-\pi,\pi]$. Beskou die Hilbertruimte $L_2[-\pi,\pi]$. Show that $\cos(x) \in L_2[-\pi,\pi]$, i.e. show that $\|\cos(x)\| < \infty$. Toon aan dat $\cos(x) \in L_2[-\pi,\pi]$, d.i. toon aan dat $\|\cos(x)\| < \infty$. Find nontrivial functions $f, g \in L_2[-\pi,\pi]$ such that (Vind nie-triviale funksies $f, g \in L_2[-\pi,\pi]$ sodat)

$$\langle f(x), \cos(x)\rangle = 0, \qquad \langle g(x), \cos(x)\rangle = 0$$

and (en)

$$\langle f(x), g(x)\rangle = 0\,.$$

2) Consider the Hilbert space $\mathbf{R}^4$. Beskou die Hilbertruimte $\mathbf{R}^4$. Find all pairwise orthogonal vectors (column vectors) $\mathbf{x}_1,\ldots,\mathbf{x}_p$, where the entries of the column vectors can only be $+1$ or -1. Vind alle paarsgewyse ortogonale vektore (kolom vektore) $\mathbf{x}_1,\ldots,\mathbf{x}_p$, waar die inskrywings van die kolomvektore slegs $+1$ of -1 kan wees. Calculate the matrix (Bereken die matriks)

$$\sum_{i=1}^{p} \mathbf{x}_i \mathbf{x}_i^T$$

and find the eigenvalues and eigenvectors of this matrix (en vind die eiewaardes en eievektore van hierdie matriks).

3) Show that (Toon aan dat)

$$s_n = \sum_{j=1}^{n} \frac{1}{(j-1)!}$$

is a Cauchy sequence (ń Cauchy reeks is).

4) Consider the Hilbert space $\mathbf{R}^4$ and the vectors (Beskou die Hilbertruimte $\mathbf{R}^4$ en die vektore)

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

a) Show that the vectors are linearly independent. Toon aan dat die vektore lineêr onafhanklik is.

b) Use the Gram-Schmidt orthogonalization process to find mutually orthogonal vectors. Gebruik die Gram-Schmidt ortogonaliseringsproses om onderling ortogonale vektore te vind.