CLASSICAL AND QUANTUM COMPUTING
EXERCISE VI

1. The number of ways of selecting k objects from n distinct objects is given by
$$\binom{n}{k} := \frac{n!}{k!(n-k)!}.$$
We have
$$\binom{n}{0} = \binom{n}{n} = 1.$$
Using this base case, and the relation
$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$
implement a recursive function to calculate $\binom{n}{k}$. Use a C++ template function, applying the function using the basic data type `unsigned long` and the abstract data type `Verylong`. Alternatively, apply the function using the data types `long` and `BigInteger` in Java. Also give an iterative implementation. Compare the execution times (time complexity) of the two implementations. Explain any differences in the execution times.

2. The Taylor series expansion at $x = 0$ of $(1+x)^{\frac{1}{4}}$ for $x^2 \leq 1$ is given by
$$(1+x)^{\frac{1}{4}} = 1 + \frac{1}{4}x - \frac{1 \cdot 3}{4 \cdot 8}x^2 + \frac{1 \cdot 3 \cdot 7}{4 \cdot 8 \cdot 12}x^3 - \frac{1 \cdot 3 \cdot 7 \cdot 11}{4 \cdot 8 \cdot 12 \cdot 16}x^4 + \ldots$$
Determine the relation between consecutive terms in the series expansion. Use the relation to implement the expansion recursively and iteratively up to the 10th term in the sum. Determine the number of multiplications performed using the formula given above explicitly and the number of multiplications when the relation is used. In each case express the answer in terms of n, the number of terms used in the expansion.