CLASSICAL AND QUANTUM COMPUTING
EXERCISE XVI

1. Sarkovskii's theorem describes an ordering of the natural numbers according which periodicities imply other periodicities for continuous maps on $\mathbf{R}$. The ordering is as follows

$$3 \triangleright 5 \triangleright 7 \triangleright 9 \triangleright \ldots \triangleright 2\cdot 3 \triangleright 2\cdot 5 \triangleright 2\cdot 7 \triangleright \ldots \triangleright 2^2\cdot 3 \triangleright 2^2\cdot 5 \triangleright 2^2\cdot 7 \triangleright \ldots \triangleright 2^3 \triangleright 2^2 \triangleright 2 \triangleright 1$$

   In other words, all odd numbers excluding 1 come first followed by the same sequence multiplied by 2 then 2^2 and so in. Lastly the powers of 2 in decreasing order.

   Write a program which can determine for any two integers x and y whether $x \triangleright y$ or $y \triangleright x$.

2. Let n be a natural number. The recursive relation used to determine the Farey fraction x_k/y_k is given by

$$x_{k+2} = \left\lfloor \frac{y_k + n}{y_{k+1}} \right\rfloor x_{k+1} - x_k$$
$$y_{k+2} = \left\lfloor \frac{y_k + n}{y_{k+1}} \right\rfloor y_{k+1} - y_k$$

   where the initial conditions are $x_0 = 0$, $y_0 = x_1 = 1$ and $y_1 = n$. The sequence of x_k/y_k is called the Farey sequence. The floor of a denoted by $\lfloor a \rfloor$ is the greatest integer which is not greater than a.

   Write a program to determine the Farey sequence for given n. Determine the first 11 elements of the sequence for $n = 5$.