Trapezium Rule - Questions

1. Evaluate $\int_1^{1.8} f(x)dx$ using the Composite Trapezium Rule, given the data

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$f(x)$	1.543	1.668	1.811	1.971	2.151	2.352	2.577	2.828	3.107

   with (a) $h = 0.1$, (b) $h = 0.2$ and (c) $h = 0.4$.

2. Determine
$$\int_0^{\pi/2} \frac{\cos x}{1+x} dx$$
   to an accuracy of $\varepsilon = 10^{-2}$, using the Composite Trapezium Rule.

3. Use the Composite Trapezium Rule to find
$$P = \int_{100}^{200} \frac{dx}{\ln x},$$
   where P is (approximately) the number of prime numbers between 100 and 200.

4. Use the Composite Trapezium Rule to find the area enclosed by $y = e^x$ and $y = 20 \ln x$, assuming that these two curves intersect at $(1.1759, 3.2410)$ and $(3.1268, 22.8009)$. Impose an accuracy of $\varepsilon = 10^{-1}$.

5. Apply the Composite Trapezium Rule to calculate $\text{erf}\,(1)$, accurate to 10^{-3}, where
$$\text{erf}\,(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

6. How many steps (n) and what step size (h) are required to determine
$$\int_{-1}^{1} e^{-x^2} dx$$
   using the Composite Trapezium Rule, to an accuracy of $\varepsilon = 0.015$?

7. Determine
$$\int_{-1}^{1} xe^x dx$$
using the Composite Trapezium Rule, to an accuracy of $\varepsilon = 0.05$.

8. Show that the Trapezium Rule is exact for all polynomials of degree 1 or less.

9. Determine
$$\int_{0}^{\pi/2} \frac{\sin x}{1+x^2} dx$$
to an accuracy of $\varepsilon = 2 \times 10^{-2}$, using the Composite Trapezium Rule. Assume that $\frac{d^3}{dx^3}\left(\frac{\sin x}{1+x^2}\right)$ has a root at $x = 0.4114$.

10.