RK2 Method - Questions

1. Use third-order (in h) expansions to determine the local error in RK2.

2. For the problem
$$\frac{dy}{dx} = x + y + xy \equiv f(x, y)$$
determine F_y for RK2. What is
$$\lim_{h \to 0} F?$$

3. Use RK2 with $h = 0.01$ to solve
$$\frac{dy}{dx} = x + y + xy \equiv f(x, y)$$
with the initial condition $y(0) = 1$, in order to find $y(0.1)$. Assuming that the local error in RK2 is given by
$$\varepsilon_{i+1} = -\frac{h^3}{6} y'''(\xi) \qquad \xi \in (x_i, x_{i+1})$$
estimate an upper bound for the global error at $x = 0.1$.

4. Applying RK2 to
$$\frac{dy}{dx} = xy$$
with the initial condition $y(1) = 1$, results in the table

i	0	1	2	3	4	5
x_i	1	1.1	1.2	1.3	1.4	1.5
y_i	1	A	B	1.41094	C	1.86528

Use RK2 to determine A, B and C. Assuming that the local error in RK2 is given by
$$\varepsilon_{i+1} = \frac{h^3}{6} y'''(\xi) \qquad \xi \in (x_i, x_{i+1})$$
estimate an upper bound for the global error at $x = 1.5$.

5.