Numerical Differentiation - Questions

1. Derive
$$y_i''' = \frac{y_{i+2} - 2y_{i+1} + 2y_{i-1} - y_{i-2}}{2h^3} + O\left(h^2\right)$$
and
$$y_i' = \frac{8y_{i+1} - y_{i+2} + y_{i-2} - 8y_{i-1}}{12h} + O\left(h^4\right).$$

2. Derive
$$y_i'' = \frac{y_{i+2} - 2y_{i+1} + y_i}{h^2} + O\left(h\right)$$
and
$$y_i' = \frac{4y_{i+1} - y_{i+2} - 3y_i}{2h} + O\left(h^2\right).$$

3. Derive
$$y_i' = \frac{y_i - y_{i-1}}{h} + O\left(h\right)$$
and
$$y_i' = \frac{y_{i-2} - 4y_{i-1} + 3y_i}{2h} + O\left(h^2\right).$$

4. Use the values of $y\left(x\right) = e^x$ at $x = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$ to estimate
   - **(a)** $y'\left(\frac{1}{2}\right)$ to order h^2
   - **(b)** $y''\left(0\right)$ to order h^2
   - **(c)** $y'''\left(1\right)$ to order h
   - **(d)** $y''\left(\frac{1}{2}\right)$ to order h^4
   - **(e)** $y'\left(\frac{3}{8}\right)$ to order h^2

5. Use the values of $y\left(x\right) = \sin x$ at $x = \left\{0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}\right\}$ to estimate
   - **(a)** $y''\left(0\right)$ to order h.
   - **(b)** $y'\left(\frac{3\pi}{16}\right)$ to order h^2.

6. Use the values of $y\left(x\right) = \cos x$ at $x = \left\{0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}\right\}$ to estimate
   - **(a)** $y''\left(\frac{\pi}{2}\right)$ to order h
   - **(c)** $y'\left(\frac{7\pi}{16}\right)$ to order h^2

7.