Newton's Method - Questions

1. Use Newton's Method to find

   (a) the negative root of $e^x - 3x^2 = 0$ with $x_1 = -0.464$ and $\varepsilon = 10^{-6}$.

   (b) the positive root of $e^x - 3x^2 = 0$ with $x_1 = 0.864$ and $\varepsilon = 10^{-6}$.

2. The equation
$$x^2 - 1 - \sin x = 0$$
has roots near -0.6 and 1.6. Use Newton's Method to find these roots, subject to a tolerance of $\varepsilon = 10^{-6}$.

3. Find the area enclosed by $y(x) = \cos(x)$ and $y(x) = e^{-x}$. You will need to use Newton's Method to find one of the limits of integration; impose a tolerance of $\varepsilon = 10^{-5}$.

4. The curves $y(x) = 2\sin x$ and $y(x) = \ln(x) - c$ touch each other near $x = 8$. Use Newton's Method to determine c. Impose a tolerance of $\varepsilon = 10^{-5}$.

5. Show that Newton's Method in the form
$$x_{i+1} = \frac{x_i(3 - ax_i^2)}{2}$$
allows the estimation of $\frac{1}{\sqrt{a}}$.

6. Prove that Newton's Method converges linearly to the root of $x^2 = 0$.

7. Show that if the approximation
$$f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$
is used in Newton's Method, then Newton's Method reduces to Linear Interpolation.

8. The function $y(x)$ is defined by the relation
$$\sin(xy) = y - x.$$
Find the values of x and y where $y(x)$ has a maximum. Assume the maximum lies near $x = 1$, and impose a tolerance of 10^{-3}.

9. Show analytically that Newton's Method converges linearly to the root of

$$(2x-1)^3 = 0.$$

Would you expect faster convergence than in the case of the Bisection Method, and why / why not?

10. Show analytically that Newton's method in the form

$$x_{i+1} = \frac{12x_i - 5x_i^3}{8}$$

can be used to estimate $\sqrt{0.8}$. Show that this method will converge if the initial estimate x_1 satisfies

$$\sqrt{\frac{4}{15}} < x_1 < \sqrt{\frac{4}{3}}.$$

11. Show analytically that Newton's method in the form

$$x_{i+1} = \frac{9x_i - 4x_i^3}{6}$$

can be used to estimate $\sqrt{0.75}$. Use this form of Newton's method, with $x_1 = 1$, to estimate $\sqrt{0.75}$ subject to a tolerance of $\varepsilon = 10^{-4}$.