Linear Interpolation - Questions

1. The equation
$$x^3 - x + 1 = 2x^2$$
has a negative root in $\left[-1, -\sqrt{\frac{1}{3}}\right]$ and a positive root in $\left[0, \sqrt{\frac{1}{3}}\right]$. Use Linear Interpolation to find these roots, subject to tolerance of $\varepsilon = 10^{-6}$.

2. Use Linear Interpolation to find the negative root of $y(x) = e^x - x - 2 = 0$ with $x_1 = -\sqrt{2}$ and $x_2 = -\sqrt{3}$, subject to a tolerance of $\varepsilon = 10^{-5}$.

3. Implement Linear Interpolation to solve
$$y(c) = c\cosh\frac{45}{c} - 15 - c = 0$$
assuming $c_1 = 67.5$ and $c_2 = 70$, subject to a tolerance of $\varepsilon = 10^{-3}$.

4. Apply Linear Interpolation to the sin function, with $x_1 = 2$ and $x_2 = 4$, to estimate π. Impose a tolerance of $\varepsilon = 10^{-5}$.

5. Apply the Linear Interpolation method to approximate the root of $f(x) = \cos x - \sin x$ using $x_1 = 0.7$ and $x_2 = 0.8$. Impose a tolerance of $\varepsilon = 10^{-9}$.

6. Show that $\sqrt[3]{a}$ can be estimated using Linear Inerpolation in the form
$$x_{i+1} = \frac{x_i x_{i-1}\left(x_i + x_{i-1}\right) + a}{\left(x_i^2 + x_i x_{i-1} + x_{i-1}^2\right)}.$$

7. Use Linear Interpolation with $x_1 = 2$ and $x_2 = 3$ to estimate the value of e. Impose a tolerance of $\varepsilon = 10^{-3}$. Show all calculations. Work to a precision of at least 6 decimal places throughout. (HINT: construct a logarithmic equation for which e is the root).

8. Show that Linear Interpolation solves any linear equation of the form $mx + c = 0$ exactly, where m and c are real constants.

9. Use Linear Interpolation with $x_1 = 3$ and $x_2 = 4$ to solve
$$\sin x = 0$$
subject to a tolerance of $\varepsilon = 10^{-2}$. Show all calculations. Work to a precision of at least 5 decimal places throughout. (NB: angles are measured in radians.)

10. Show analytically that Linear Interpolation in the form

$$x_{i+1} = \frac{x_{i-1}x_i + 0.8}{x_{i-1} + x_i}$$

can be used to estimate $\sqrt{0.8}$. Use this method to estimate $\sqrt{0.8}$, with $x_1 = 0$ and $x_2 = 1$. Impose a tolerance of $\varepsilon = 10^{-3}$ on the problem. Work to a precision of at least 4 decimal places throughout.

11.