Lagrange Interpolation - Questions

1. Given the data

i	0	1	2	3
x_i	7	8	9	10
y_i	3	1	1	9

   estimate $y\,(9.5)$ using a Lagrange interpolating polynomial.

2. Consider the exponential function $y(x) = e^x$. Use a Lagrange interpolating polynomial to estimate $e^{0.2}$, using $y(x)$ at the nodes $\{0, 0.1, 0.3\}$. Estimate the maximum error in your approximation to $e^{0.2}$.

3. Given the data

i	0	1	2	3	4
x_i	1	2	3	4	5
y_i	11.6	16.2	16.8	13.5	7.3

   estimate the position and value of the maximum in $y\,(x)$, using an appropriate Lagrange interpolating polynomial.

4. Determine the total number of multiplication and division operations to interpolate n points using a Lagrange interpolating polynomial.

   (a) Find the Taylor expansion to fourth order of $\cos x$ about $x = 0$.

   (b) Find the Lagrange interpolating polynomial for the data

i	0	1	2	3	4
x_i	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos x_i$	0	$\frac{1}{2}$	1	$\frac{1}{2}$	0

   (c) Find upper bounds on the error for the Taylor polynomial in (a) and the Lagrange interpolating polynomial in (b) on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

5. Construct a polynomial that interpolates $y\,(x) = e^x \sin x$ at the nodes $\left\{-2, -\frac{5}{4}, -\frac{1}{2}, \frac{1}{4}, 1\right\}$. Obtain a polynomial expression for the upper bound on the approximation error $\Delta\,(x)$.

6. Assume that the function

$$f(x) = \frac{1}{1+x^2}$$

is approximated by means of a Lagrange interpolating polynomial, using the nodes $\{-1, 0, 1\}$. Estimate the approximation error at $x = -0.8$ and $x = 0.5$, **without** determining the interpolating polynomial.

7. Determine the Lagrange polynomial $P(x)$ that interpolates

$$y(x) = e^{-x^2} - \frac{1}{e}$$

at the nodes

$$x_i = \left\{-1, -\frac{1}{3}, \frac{1}{3}, 1\right\}.$$

Estimate $y(0.6)$ using $P(x)$. Use

$$\max_{[-1,1]} \left|y^{(4)}(x)\right|$$

to determine an upper bound for the error in $P(0.6)$. Compare this upper bound to the actual error.

$$\left[\begin{array}{l} \text{HINT}: \\ y^{(4)} = e^{-x^2}\left(12 - 48x^2 + 16x^4\right) \\ 32x^5 - 160x^3 + 120x \text{ has roots at } \pm 0.9586 \text{ and } 0 \text{ on } [-1,1] \end{array}\right]$$

8. Determine the Lagrange polynomial $P(x)$ that interpolates

$$y(x) = x \sin x - 3 \sin x$$

at the nodes

$$x_i = \{0, 1, 2, 3\}.$$

Estimate $y\left(\frac{3}{2}\right)$ using $P(x)$.

Use

$$\max_{[0,3]} \left|y^{(4)}(x)\right|$$

to determine an upper bound for the error in $P\left(\frac{3}{2}\right)$.

$$\left[\text{HINT}: y^{(4)} = y - 4\cos x \quad \text{and} \quad x + 5\tan x = 3 \Rightarrow x \approx 0.46865\right]$$

Compare this upper bound to the actual error.

9.