

Fixed Point - Questions

1. The equation

$$e^x - 3x^2 = 0$$

has roots near $-\frac{1}{2}$, 1 and 4. Write this equation in three ways that facilitate Fixed Point iteration, and test the convergence of each to each of the three roots.

2. The equation

$$x^2 - 10 \ln(x) = 0$$

has roots near 1.1384 and 3.5656. Write this equation in three ways that facilitate Fixed Point iteration, and test the convergence of each to each of the two roots.

3. The equation

$$x \cos x = x \sin x$$

has a root at $x = \frac{\pi}{4}$. Which iteration process

$$x_{j+1} = x_j \tan x_j$$

$$x_{j+1} = x_j \cot x_j$$

should be used to find this root?

4. We seek a non-zero solution of $\sin(x^2) = 0$ using the Fixed Point method. Show that we can use the equation

$$x = \frac{\sin(x^2)}{x^2} + x$$

which has the same non-zero roots as $\sin(x^2) = 0$. Show that we expect Fixed Point iteration to converge to the root at $\sqrt{\pi}$. Apply Fixed Point iteration starting at $x = 2$, and impose a tolerance of 10^{-4} on $f(x) \equiv \sin(x^2)$. Could we also use

$$x = \frac{\sin(x^2)}{x} + x$$

to find the root at $\sqrt{\pi}$?