Euler's Method - Questions

1. Use a Taylor expansion of $y(x_{i+1})$ to determine the local error in Euler's Method.

2. Use Euler's method with $h = 0.01$ to solve

$$\frac{dy}{dx} = x + y + xy \equiv f(x, y)$$

with the initial condition $y(0) = 1$, in order to find $y(0.1)$. Assuming that the local error in Euler's Method is given by

$$\varepsilon_{i+1} = -\frac{h^2}{2} y''(\xi) \qquad \xi \in (x_i, x_{i+1})$$

estimate an upper bound for the global error at $x = 0.1$.

3. Solve

$$\frac{dy}{dx} = \frac{x}{y} \equiv f(x, y)$$

with the initial condition $y(0) = 1$, using Euler's Method with $h = 0.1$, up to $x = 0.9$. Estimate an upper bound on the global error in y_5 and y_9, assuming that the local error in Euler's Method is given by

$$\varepsilon_{i+1} = -\frac{h^2}{2} y''(\xi) \qquad \xi \in (x_i, x_{i+1}).$$

4. Applying Euler's method to

$$\frac{dy}{dx} = x + 2y + xy$$

with the initial condition $y(0) = 2$, results in the table

i	0	1	2	3	4	5
x_i	0	0.01	0.02	0.03	0.04	0.05
y_i	2	A	B	2.1233	2.1667	2.2113

i	6	7	8	9	10
x_i	0.06	0.07	0.08	0.09	0.10
y_i	2.2572	C	2.3527	D	2.4535

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Use Euler's method to determine A, B, C and D. Assuming that the local error in Euler's method is given by

$$\varepsilon_{i+1} = -\frac{h^2}{2} y''(\xi) \qquad \xi \in (x_i, x_{i+1})$$

estimate an upper bound for the global error at $x = 0.08$.

5. Applying Euler's method to

$$\frac{dy}{dx} = x + y + xy$$

with the initial condition $y(0) = 1$, results in the table

i	0	1	2	3	4	5
x_i	0	0.01	0.02	0.03	0.04	0.05
y_i	1	1.0100	A	1.0309	B	1.0531

i	6	7	8	9	10
x_i	0.06	0.07	0.08	0.09	0.10
y_i	1.0646	C	D	1.1013	1.1142

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Use Euler's method to determine A, B, C and D. Assuming that the local error in Euler's method is given by

$$\varepsilon_{i+1} = -\frac{h^2}{2} y''(\xi) \qquad \xi \in (x_i, x_{i+1})$$

estimate an upper bound for the global error at $x = 0.1$.

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