Chebyshev Polynomials - Questions

1. Determine the Chebyshev polynomials $T_0(x)$ up to $T_{10}(x)$.

2. Write the powers of x, from x^0 up to x^{10}, in terms of Chebyshev polynomials.

3. Find the zero points and the extreme points of
$$T_5(x) = 16x^5 - 20x^3 + 5x.$$

4. Economize
$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!}$$
by writing
$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + \frac{x^6}{6!}$$
in terms of Chebyshev polynomials, up to $T_5(x)$.

5. Economize
$$\sin(x) = x - \frac{x^3}{3!}$$
by writing
$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$$
in terms of Chebyshev polynomials, up to $T_3(x)$.

6. Economize
$$\cos(x) = 1 - \frac{x^2}{2!}$$
by writing
$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$$
in terms of Chebyshev polynomials, up to $T_2(x)$.

7. Determine the coefficients in the expansion
$$\arccos(x) = \frac{1}{2}c_0 + \sum_{k=1}^{\infty} c_k T_k(x)$$
by exploiting the orthogonality of the Chebyshev polynomials.

8. Expand the step function

$$f(x) = \begin{cases} 1 & x \geqslant 0 \\ 0 & x < 0 \end{cases}$$

in terms of Chebyshev polynomials.

9. Prove that

$$T_m(T_n(x)) = T_{mn}(x).$$

10. Prove that

$$T_{m+n}(x) + T_{m-n}(x) = 2T_m(x)\,T_n(x)\,.$$