



APPLIED MATHEMATICS 3B

Semester Test: 4 October 2011

Duration: 80 minutes

Marks: 30

- Instructions:** Answer all the questions
All calculations must be shown
Pocket calculators are permitted
All angles are measured in radians
The prescribed text book is allowed

Question 1

Consider the Hilbert space $\mathbb{C}^4 \equiv \mathbb{C}^2 \otimes \mathbb{C}^2$. We prepare a classical mixture of 75% of the Bell state $(1, 0, 0, 1)^T / \sqrt{2}$ and 25% of the Bell state $(0, 1, 1, 0)^T / \sqrt{2}$.

- (a) Find the density matrix ρ that models this mixture. (2)
(b) Calculate the partial traces $\text{tr}_1 \rho$ and $\text{tr}_2 \rho$ where the first system is \mathbb{C}^2 and the second system is \mathbb{C}^2 . (5)
(c) Use the von Neumann entropy measure to determine the amount of entanglement of ρ . (3)
(10)

Question 2

The NOT operation given by the table

a	NOT(a)
0	1
1	0

is obviously reversible. Let $\{|0\rangle, |1\rangle\}$ denote an orthonormal basis in \mathbb{C}^2 .

- (a) Give the bra-ket representation of U_{NOT} acting on \mathbb{C}^2 defined by

$$U_{NOT}|0\rangle = |1\rangle, \quad U_{NOT}|1\rangle = |0\rangle.$$

Find the eigenvalues and eigenvectors of U_{NOT} . (3)

- (b) We can also consider a unitary operator V_{NOT} acting on \mathbb{C}^4 defined by

$$V_{NOT}(|a\rangle \otimes |0\rangle) = |a\rangle \otimes |\text{NOT}(a)\rangle, \quad a \in \{0, 1\}.$$

Give a bra-ket representation of V_{NOT} .

Find the eigenvalues and eigenvectors of V_{NOT} .

What is the relation between U_{NOT} and V_{NOT} ? (7)

(10)

Question 3

Find the singular value decomposition of the matrix

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 2 & 1 \end{pmatrix}.$$

(10)

END OF QUESTION PAPER