UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF APPLIED MATHEMATICS

MODULE **APM3B10**
APPLIED MATHEMATICS 3B

CAMPUS **APK**

EXAM **NOVEMBER 2011**

DATE: 8/11/2011 **SESSION:** 09:00 – 12:00

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EXTERNAL MODERATOR DR. F. SOLMS

DURATION: 3 HOURS **MARKS:** 40

NUMBER OF PAGES: 2 PAGES

INSTRUCTIONS: ANSWER ALL THE QUESTIONS

ALL CALCULATIONS MUST BE SHOWN

POCKET CALCULATORS ARE PERMITTED

ALL ANGLES ARE MEASURED IN RADIANS

THIS IS AN **OPEN BOOK** EXAM

QUESTION 1

Consider the state

$$\boldsymbol{\psi} := \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

where $\theta, \phi \in \mathbb{R}$. Construct the density matrix $\rho(0) := \boldsymbol{\psi}\boldsymbol{\psi}^*$.

Given the Hamilton operator $\hat{H} = \hbar\omega\sigma_y$, $\sigma_y := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\omega \in \mathbb{R}$, solve the von Neumann equation for $\rho(t)$.

(10)

QUESTION 2

Consider the state

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle).$$

We wish to teleport the *second* qubit of this entangled pair, i.e. we prepare the state

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle).$$

Apply the teleportation protocol on the right hand three qubits. Describe every step in detail.
Are the first and teleported qubits still entangled? **(10)**

QUESTION 3

Let $\{\, |0\rangle, |1\rangle \,\}$ be an orthonormal basis for $\mathbb{C}^2$. Use the singular value decomposition to determine whether

$$\frac{1}{2}(|0\rangle \otimes |0\rangle + |0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle)$$

is entangled, and to find a representation of the state with a minimum number of tensor products. **(10)**

QUESTION 4

Let

$$\{\, |0\rangle, |1\rangle, |2\rangle, |3\rangle \,\}$$

denote an orthonormal basis in $\mathbb{C}^4$. Apply the quantum Fourier transform on $\mathbb{C}^4$ to the state

$$|\psi\rangle := \frac{1}{2}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle + |2\rangle \otimes |0\rangle + |3\rangle \otimes |1\rangle)$$

i.e. apply $U_{QFT,4} \otimes I_4$. The quantum Fourier transform on $\mathbb{C}^4$ is given by

$$U_{QFT,4} = \frac{1}{2}\sum_{j=0}^{3}\sum_{k=0}^{3} e^{-i2\pi jk/4}|j\rangle\langle k|.$$

Use your answer to analyze the periodicity of the sequence of states in $|\psi\rangle$ (0101). **(10)**

END OF QUESTION PAPER