Applied Mathematics 3B

Assignment #9 **7:30, 18 October 2011**

---

1. The discrete Fourier transform over n points can be written in matrix form

$$F_n := \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{n-1} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{pmatrix}$$

where $w = e^{2\pi i/n}$ is the n-th root of unity. We obtain the discrete Fourier transform from

$$(\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)^T = F_n(x_1, x_2, \dots, x_n)^T.$$

Calculate $F_4(0,1,0,1)^T$ and $F_4(-1,1,-1,1)^T$. Interpret the results to find the underlying periodicity.

2. The discrete Fourier transform over n points can be written in matrix form

$$F_n := \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & w & w^2 & \dots & w^{n-1} \\ 1 & w^2 & w^4 & \dots & w^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \dots & w^{(n-1)^2} \end{pmatrix}$$

where $w = e^{2\pi i/n}$ is the n-th root of unity. Show that the inverse F_n^{-1} is given by

$$F_n^{-1} := \frac{F_n^*}{n} = \frac{1}{n} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & (w^*) & (w^*)^2 & \dots & (w^*)^{n-1} \\ 1 & (w^*)^2 & (w^*)^4 & \dots & (w^*)^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & (w^*)^{n-1} & (w^*)^{2(n-1)} & \dots & (w^*)^{(n-1)^2} \end{pmatrix}$$

where $w^* = e^{-2\pi i/n}$.

3. Refer to chapter 15, problem 1 in the textbook

Problems and Solutions in Quantum Computing and Quantum Information, 2nd edition.

Use the method described to find the two nontrivial factors of $n = 35$ using $a = 3$.
Can the method be applied to find the factors of 35 when $a = 11$?