UNIVERSITY OF JOHANNESBURG

Applied Mathematics 3B

Assignment #6 **7:30, 20 September 2011**

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1. Find a unitary operator U_1 that implements the boolean constant function 1:

$$f_1 : \{0,1\} \to \{0,1\}, \qquad f_1 : x \mapsto 1.$$

What is the minimum number of qubits required to represent the input (domain) and output (range) of f_1?

Find a unitary operator U_{AND} that implements the boolean function f_{AND}:

$$f_{AND} : \{0,1\}^2 \to \{0,1\}, \qquad f_{AND} : (x,y) \mapsto xy.$$

What is the minimum number of qubits required to represent the input (domain) and output (range) of f_{AND}?

2. Refer to chapter 3, problem 6 in the textbook

Problems and Solutions in Quantum Computing and Quantum Information, 2nd edition.

Find the singular value decompositions of

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 3 \end{pmatrix}.$$

3. The U_{CNOT} gate is defined on page 112 problem 21 in the textbook

Problems and Solutions in Quantum Computing and Quantum Information, 2nd edition.

In braket notation we have

$$U_{CNOT} := |0\rangle\langle 0| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|) + |1\rangle\langle 1| \otimes (|0\rangle\langle 1| + |1\rangle\langle 0|).$$

Find the eigenvectors of U_{CNOT}.

Hint: The eigenvectors $|\psi\rangle$ of U_{CNOT} can be written in the form

$$|\psi\rangle = a|0\rangle \otimes |0\rangle + b|0\rangle \otimes |1\rangle + c|1\rangle \otimes |0\rangle + d|1\rangle \otimes |1\rangle$$

where $a, b, c, d \in \mathbb{C}$, and the eigenvalues can be written in the form $e^{i\theta}$.